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PID- Sliding mode and Lyapunov design controllers for power optimization of variable speed wind turbines

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Abstract: *Two Non linear control strategies are proposed for two different conditions unlike those presented in our previous work. Sliding mode with PID controller is proposed as a robust controller for the case when wind speed measurement is available while Lyapunov based controller is designed for the case when effective wind speed measurement is not available. The main objective of the controllers is optimization of wind energy captured while avoiding strong transients in the turbine components that may reduce mechanical lifespan of the turbine. The controllers are first tested with simplified mathematical model and then validated upon a flexible wind turbine simulator for high-turbulence wind speed profile. The two proposed control strategies are compared and results obtained show good performance.*

Keywords: Variable speed wind turbine, Sliding mode control, Lyapunov design based controller, Power capture optimization.

1. Introduction

As a result of increasing environmental concern and rising fossil fuel prices along with energy demand, more and more electricity is being generated from renewable sources. Wind energy conversion systems have quickly evolved over the last decades, therefore, efficient and reliable exploitation tools are necessary to make these installations more profitable [13]. Variable speed wind turbines (VSWT) show many advantages compared to former fixed speed ones. The annual production of a VSWT exceeds by 5 to 10% over a fixed speed ones [12]. Effects of wind power fluctuations can also be attenuated using this kind of turbines. However, it was shown that the control strategy has a major impact on the WT behavior and on the loads transmitted to the network [3], and that whatever the WT kind, the control system remains a key factor [4].

Many contributions have been devoted to the control of the aeroturbine mechanical as well as the electrical components. Some of them are primarily based on linear time-invariant (LTI) models. Classical controllers have also been used extensively. Optimal control has been applied in the linear quadratic (LQ) [1, 11] and linear quadratic Gaussian (LQG) [9, 11] forms. Robust control was introduced in [5, 10]. More recently some non linear control laws have been proposed [2, 6- 8].
The main control objective is optimizing the extracted aerodynamic power in partial load area. In this paper, two nonlinear control strategies are proposed for robust operation of VSWT. The controllers are designed for two cases, one when effective wind speed measurement is available (SMCPID) and other when it is not available (Lyapunov design based controller. [7, 8].

This paper is organized as follows: Section 2 describes the wind turbine modelling, the control objectives are then briefly exposed. Section 3 describes the two proposed control strategies: Sliding mode control–PID controller and the Lyapunov design based controller. In Section 4, validation results on wind turbine simulator showing the performance of the proposed approaches are included.

2. Wind turbine modeling and control objectives

A Wind turbine modelling

The aerodynamic power captured by the wind turbine depends on wind velocity v, the power coefficient of the machine C_p and change in rotor speed with respect to wind.

$$P_a = \frac{\rho}{2}\pi R^2 C_p(\lambda,\beta)v^3 \quad (1)$$

Tip-speed ratio λ is given by

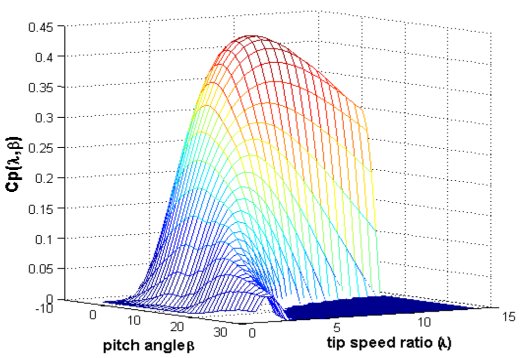
$$\lambda = \frac{\omega_t R}{v} \quad (2)$$

Where ω_t is the rotor speed, R is the rotor radius, and ρ the air density. The power coefficient $C_p(\lambda,\beta)$ is a non linear function of λ and blade pitch angle β as shown in figure 1(a). From the relation $P_a = T_a.\omega_t$, the expression for T_a is

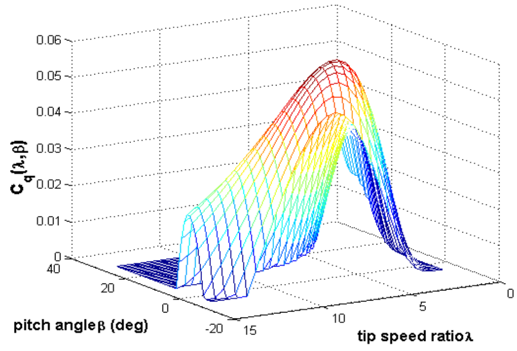
$$T_a = \frac{\rho}{2}\pi R^3 C_q(\lambda,\beta)v^2 \quad (3)$$

$$C_q(\lambda,\beta) = \frac{C_p(\lambda,\beta)}{\lambda} \quad (4)$$

$C_q(\lambda,\beta)$ is the torque coefficient and its plot is shown in figure 1(b). The torque and speed curve are obtained using the blade element theory, evaluated by a code developed by the National Renewable Energy Laboratory.



(a) : Power coefficient $C_p(\lambda,\beta)$ curve



(b) : Torque coefficient $C_q(\lambda,\beta)$

Figure 1: Wind turbine characteristics

If a perfectly rigid low-speed shaft is assumed, a single mass model of the turbine may then be considered [6]

$$J_t\dot{\omega}_t = T_a - k_t\omega_t - T_g \quad (5)$$

where

$$J_t = J_r + n_g^2 J_g \quad (6)$$

$$K_t = K_r + n_g^2 K_g \quad (7)$$

$$T_g = n_g T_{em} \quad (8)$$

The one mass wind turbine model is shown in figure 2.

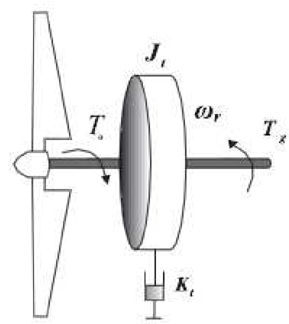


Figure 2 : One mass model of wind turbine

B Control objectives

The objective of the aero turbine controller is to optimize wind power capture. The aerodynamic power captured by the aero turbine rotor given by eq (1) is maximum, when $C_p(\lambda,\beta)$ is at its unique maximum C_{popt}. For a given wind speed

$$C_p(\lambda_{opt},\beta_{opt}) = C_{popt} \quad (9)$$

In order to maintain λ at its optimum value for a given speed v, the rotor speed must be adjusted using the generator torque to track the reference

$$\omega_{topt} = \frac{\lambda_{opt}}{R}v \quad (10)$$

The blade pitch angle β is fixed to its optimal value β_{opt}.

The aim of the controller is to track this optimal rotor speed ω_{topt} while trying to reduce control stress. Robustness of the controller and effectiveness of the controller when wind speed measurement is not available are also very important.

3. Control strategies

Two control strategies are herein proposed. First a Sliding mode with PID (SMCPID) controller and second a Lyapunov design based controller.

A Sliding mode with PID controller

The one mass model of variable speed wind turbine is

$$J_t\dot{\omega}_t = T_a - k_t\omega_t - T_g$$

By taking f and g as

$$f = \frac{T_a}{J_t} - \frac{k_t\omega_t}{J_t} \tag{11}$$

$$g = \frac{1}{J_t} \tag{12}$$

We get

$$\dot{\omega}_t = f(\omega_t) + g(\omega_t)u \tag{13}$$

Sliding surface s is defined as

$$s = \omega_t - \omega_{opt} \tag{14}$$

As the relative degree of the system is 1

$$\dot{s} = f - g.T_g - \dot{\omega}_{opt} \tag{15}$$

Let us define

$$\hat{f} = \frac{\hat{T}_a}{\hat{J}_t} - \frac{\hat{k}_t\omega_t}{\hat{J}_t} \tag{16}$$

$$\delta = \frac{\hat{J}_t}{J_t} \tag{17}$$

Based on reachability condition of sliding mode control

$$S^T\dot{S} < -\mu\|S\|_2 \tag{18}$$

$$T_g = \hat{J}_t\left(\{\hat{f} - \dot{\omega}_{opt}\} + as + ksign(s) + k_ps + k_i\int sdt + k_d\frac{ds}{dt}\right) \tag{19}$$

Equation (11) can be written as

$$\dot{s} = \left(f - \delta\hat{f}\right) - (1-\delta)\dot{\omega}_{opt} - \delta as - \delta\left(ksign(s) + k_ps + k_i\int sdt + k_d\frac{ds}{dt}\right) \tag{20}$$

The reachability condition given in equation (18) is satisfied for values of k

$$k \geq \left|\frac{(f-\delta\hat{f})}{\delta} - \frac{(1-\delta)}{\delta}\dot{\omega}_{opt} - as\right| + \frac{\mu}{\delta} \tag{21}$$

Assuming,

$$\left|f - \hat{f}\right| \leq \eta \tag{22}$$

The control law ensures a robust tracking of the optimal rotor speed for all k

$$k \geq \frac{\eta}{\delta} + \frac{\mu}{\delta} + \left|\frac{(1-\delta)}{\delta}(\hat{f} - \dot{\omega}_{opt})\right| + a|s| \tag{23}$$

The additional PID control helps in driving the system on to the sliding surface. The proportional action drives the states to the neighbourhood of the sliding surface. Integral action forces the states onto the sliding surface irrespective of the bounds of the uncertainties and disturbances, while the derivative action provides a stabilizing effect to counter the possible excessive control produced by integral action.

Assuming that the wind speed v is accessible, $\hat{T}_a$ and ω_{opt} calculated online as shown below

$$\hat{T}_a = k_{opt}\omega_{opt}^2 \tag{24}$$

$$\omega_{opt} = \frac{\lambda_{opt}v}{R} \tag{25}$$

B Lyapunov based design

From equation (5)

$$J_t\dot{\omega}_t = T_a - k_t\omega_t - T_g$$

If the rotor tracking error is defined as

$$e = \omega_t - \omega_{opt} \tag{26}$$

Then equation (5) in terms of e and ω_{opt} will be

$$\dot{e} = -\dot{\omega}_{opt} - \frac{k_t}{J_t}\left(\omega_{opt} + e\right) - \frac{T_g}{J_t} + \frac{T_a}{J_t} \tag{27}$$

T_a is considered as the unknown variable. If $\widehat{T_a}$ is the estimate of unknown T_a , then the error in estimate is given by

$$\overline{T_a} = T_a - \widehat{T_a} \tag{28}$$

If this estimate is correct $T_a = \widehat{T_a}$ then the following control law

$$T_g = J_t\left(-\dot{\omega}_{opt} - \frac{k_t}{J_t}\omega_{opt} + Ce + \frac{\hat{T}_a}{J_t}\right) \tag{29}$$

would achieve global asymptotic tracking.
But as $\overline{T_a} \neq 0$,

$$\dot{e} = -\left(\frac{k_t}{J_t} + C\right)e - \frac{\overline{T_a}}{J_t} \tag{30}$$

The parameter error $\overline{T_a}$ continues to act as a disturbance which may destabilize the system. So an update law for $\widehat{T_a}$ which preserves the boundedness of e and achieves asymptotic tracking has to be designed.

Considering the Lyapunov function as

$$V_1 = \frac{e^2}{2} + \frac{1}{2}\left(\frac{\overline{T_a}^2}{k}\right) \tag{31}$$

where k is a positive gain.

Rotor diameter	43.3 m
Gearbox ratio(n_g)	43.165
Tower height	36.6 m
Rated Power	600 kw
Maximum generator torque (T_{gmax})	162 kN.m

Table 1: CART wind turbine characteristics

The time derivative of V_1 is

$$\dot{V}_1 = -\left(\frac{\mathrm{k_t}}{\mathrm{J_t}} + \mathrm{C}\right)\mathrm{e}^2 - \frac{\overline{T_a}}{\mathrm{J_t}}\mathrm{e} + \frac{\overline{T_a}\dot{\widehat{T_a}}}{k} \quad (32)$$

$$\dot{V}_1 = -\left(\frac{\mathrm{k_t}}{\mathrm{J_t}} + \mathrm{C}\right)\mathrm{e}^2 - \frac{\overline{T_a}}{\mathrm{J_t}}\left(\mathrm{e} - \frac{\dot{T_a}}{k} + \frac{\dot{\widehat{T_a}}}{k}\right) \quad (33)$$

The stability condition

$$\dot{V}_1 < 0 \quad (34)$$

can be achieved for any unknown $\overline{T_a}$ by choosing the update law

$$\dot{\widehat{T_a}} = \xi + k\frac{e}{\mathrm{J_t}} \quad (35)$$

This leads to

$$\dot{V}_1 = -\left(\frac{\mathrm{k_t}}{\mathrm{J_t}} + \mathrm{C}\right)\mathrm{e}^2 \quad (36)$$

garanteeing global stability of the equilibrium e=0, $\overline{T_a} = 0$ and hence boundedness of ω_t and $\widehat{T_a}$.

Optimum reference speed used for tracking can be obtained by

$$\omega_{opt} = \sqrt{\frac{\widehat{T_a}}{k_{opt}}} \quad (37)$$

It is assumed that wind turbine is operating in its optimum region, hence

$$k_{opt} = \frac{\rho}{2}\pi R^5 C_{popt}\frac{1}{\lambda_{opt}^3} \quad (38)$$

For simulations, value of c is chosen as c=10; and white noise $\xi = 10^4$.

4. Validation results

The numerical simulations are performed with the parameters of Controls Advanced Research Turbine (CART) localized in NREL site nearby Colorado. CART was modeled with simplified mathematical model for simulation and with the FAST aero elastic simulator for validation.

The Fatigue, Aerodynamics, Structures and Turbulence (FAST) code developed by NREL is an aero elastic WT simulator capable of modelling two and three bladed propeller-type machines. This code is used by WT designers to predict both extreme and fatigue loads. The aerodynamic behavior is described by blade momentum theory. Variable wind field is assumed across the blades. FAST subroutines are coupled in an S-Function to be incorporated in a Simulink model. The main parameters of CART are summarized in table 1.

The full-field turbulent wind set v used in this study is generated using SNwind developed by NREL and coupled with FAST. The hub-height wind speed profile is illustrated in figure 3.

One must keep in mind that the controller's objectives are power capture optimization while avoiding strong efforts on the drive train and high-turbulent control torque. The controllers efficiency is compared using two criteria: the aerodynamic η_{aero} and the electrical efficiency η_{elec}.

$$\eta_{aero} = \frac{\int_{t_{ini}}^{t_{fin}} P_a dt}{\int_{t_{ini}}^{t_{fin}} P_{a_{opt}} dt} \quad (39)$$

$$\eta_{elec} = \frac{\int_{t_{ini}}^{t_{fin}} P_e dt}{\int_{t_{ini}}^{t_{fin}} P_{a_{opt}} dt} \quad (40)$$

Where $P_{a_{opt}}$ is the optimum aerodynamic power

$$P_{a_{opt}} = \frac{\rho}{2}\pi R^2 C_{popt} v^3 \quad (41)$$

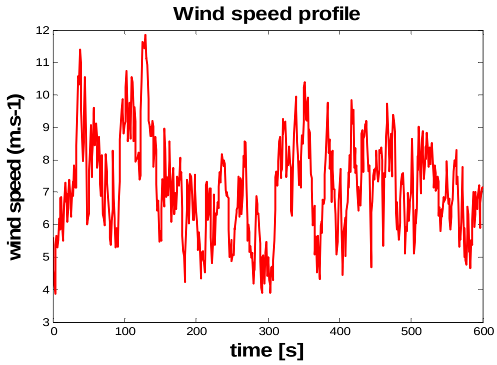


Figure 3: Wind speed profile with mean value 7 m/s at the hub-height and a turbulence intensity of 25%

Validation results are depicted in figures 4 and 5. It can be clearly seen than SMCPID is robust with respect to parameter uncertainties as rotor speed tracks its optimum value even when parameter uncertainties are 50%. Also maximum aerodynamic torque referred to high speed shaft (T_g/n_g) is much less than the allowed maximum of 3.5kN.m. Lyapunov design based controller gives good performance even if effective wind speed cannot be measured. From the comparison analysis of two proposed control strategies given in Table 2, it can be concluded that both control strategies gives good performance for different cases of operation depending upon availability of effective wind speed measurement.

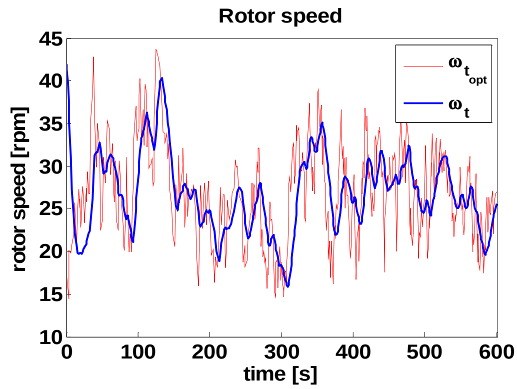


Figure 4(a): Rotor speed

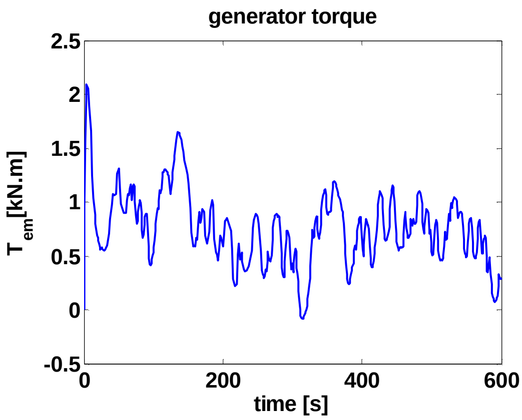


Figure 4(b): Generator torque

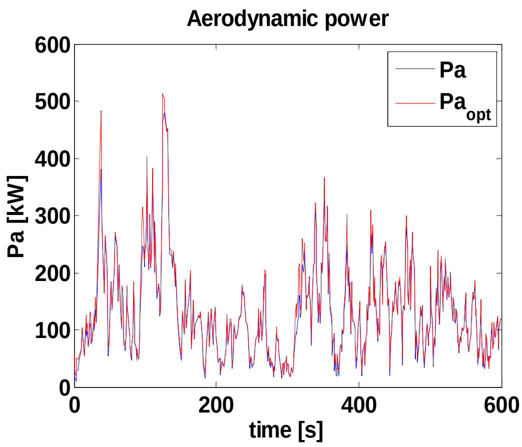


Figure 4(c): Aerodynamic power captured

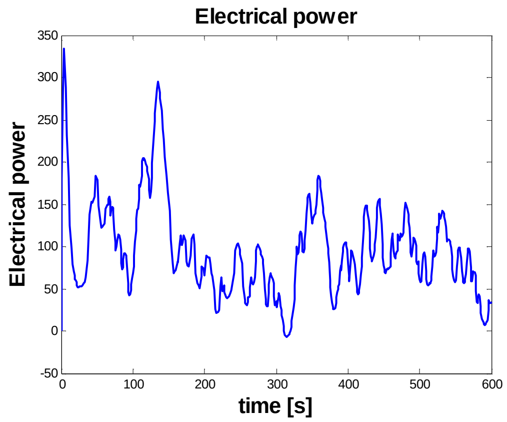


Figure 4(d): Electric power output

Figure 4: validation results of SMCPID with 50% parameter uncertainties.

Parameter	Lyapunov based Controller	SMCPID
η_{aero} (%)	92.0904	95.4297
η_{elec} (%)	64.1990	63.8084
Std(T_{ls}) (kN.m)	10.8989	11.7751
Std(T_{em}) (kN.m)	0.2346	0.3123
Max(T_{ls}) (kN.m))	82.3138	68.7312
Max(T_{em}) (kN.m)	1.8753	1.6544

Table 2: Comparison of the two control strategies

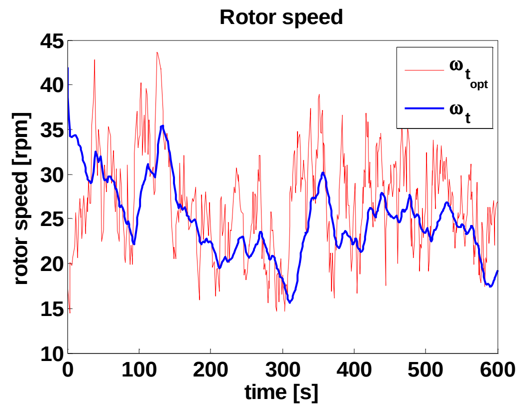


Figure 5(a): Rotor speed

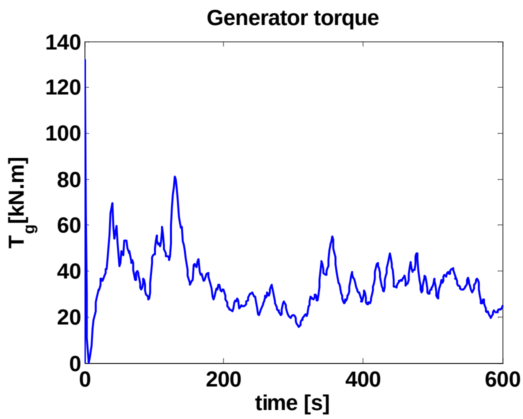


Figure 5(b): Generator torque

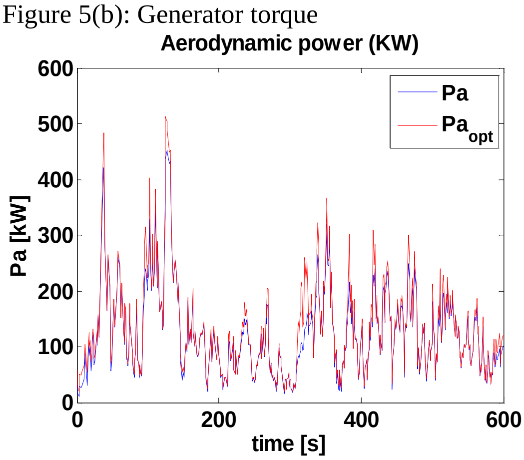


Figure 5(c): Aerodynamic power captured

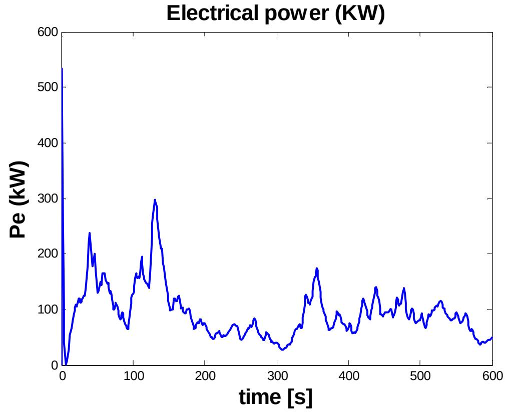


Figure 5(d): Electric power output

Figure 5: Validation results of Lyapunov design based controller

5. Conclusions

With the aim of maximizing wind power capture of a variable speed wind turbine, two non linear control strategies are proposed namely Sliding mode with PID control and Lyapunov design based controller. On the one hand, the first controller uses PID in conjunction with sliding mode control to reduce the load on the switching control to reach the sliding surface. This controller is shown to be robust with respect to parameter uncertainties provided wind speed measurement is available. On the other hand Lyapunov based controller includes a wind speed estimator whose update law is obtained using Lyapunov stability analysis. It has been shown that Lyapunov design based controller gives good performance even if effective wind speed measurement is not available.

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