NONLINEAR DYNAMIC FEEDBACK CONTROL OF A VARIABLE SPEED WIND TURBINE

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***Abstract*— Two nonlinear control laws for a variable speed wind turbine have been considered. The control objective is to maximize energy capture of the wind turbine by tracking optimal rotor rotational speed that corresponds to wind speed variations. Nonlinear Feedback Control has been first derived. Because of the lack of robustness of this approach Adaptive Nonlinear Control law has been synthesized in order to keep the required performance in spite of model parameters variations. Numerical simulation shows satisfactory results.**

***Index Terms*— wind turbine, variable speed, nonlinear control, feedback linearization, adaptive control**

I. INTRODUCTION

Adapted control systems are necessary to improve wind turbines behavior in order to make them more profitable and more reliable. The control objective depends in the region where the wind turbine (WT) operates. Control system design objectives can be specified by :

– Limitation and smoothing of electrical power in the above rated power area.
– Generation of maximum power in the below rated power area.
– Minimization of transient loads in all turbine components.

In the literature, some control strategies have been proposed generally based on LTI models about an operating point [1], [2]. Classical controllers have been extensively used, particularly the PI regulator [3], [4]. Optimal control have been used in [5]. However, the drawbacks in using linear models remain in the fact that when the wind variations are large (gusts), the turbine performance decrease.
Some nonlinear control laws have been given in [6], and adaptive control have been used in [7].
The purpose of this work is to design a nonlinear control law in order to optimize wind power capture in the partial load operation based on asymptotic desired output tracking.
This controller is different from the one suggested in [7] in terms of the generator torque expression and in addition, it is here used an adaptive control in order to robustify the optimal power tracking with respect to the wind turbine parameters variation.
The wind speed time repartition makes that, in most of time, the wind turbines are operating in wind speed less than rated one, hence the importance of control efficiency arises in this operating regime. While energy is captured from the wind, the aerodynamic power should be maximized.
The wind turbine considered in this study is a variable speed WT. The variable speed option interest comes out from the fact that it reduces stress due to the transient loads in the main shaft during the full load operation of the wind turbine and optimizes energy extraction over all wind speeds below rated. An additional benefit is that the variable speed turbines rotate far less during their life time ; i.e. they can be brought to a lower rotational speed in the low wind speed region.

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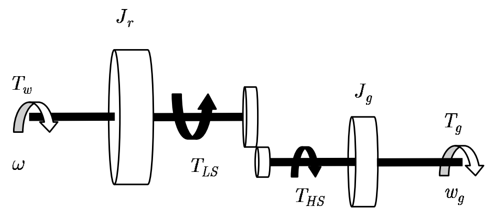


Fig. 1. Wind turbine scheme

II. Problem formulation

The objective of the controller is to maximize wind power extraction by adjusting the rotor rotational speed ω to wind speed variation such that the aerodynamic power stands at its maximum in spite of this variation. The control action is the excitation winding voltage of the asynchronous generator connected to the rotor through the gearbox.
The aerodynamic power is given by the expression

$$P_w = \frac{1}{2}\rho\pi R^2 v^3 C_p(\lambda) \tag{1}$$

where $\lambda = \dfrac{\omega R}{v}$ is the tip speed ratio, v the wind speed, R the rotor radius, ρ the air density and $C_p(\lambda)$ the power coefficient.
The $C_p(\lambda)$ curve has a unique maximum at

$$\lambda_{opt} = \frac{\omega^* R}{v_{ref}} \tag{2}$$

that corresponds to a maximum power production.
In order to make λ tracking its optimal value, the rotor speed is then adjusted to track the reference ω^* which have the same shape as wind speed reference v_{ref} since they are proportional.

$$\omega^* = \frac{\lambda_{opt}}{R} v_{ref} \tag{3}$$

So, in the control scheme, the rotor speed ω is the system output, the input is the voltage V across the generator.

III. Wind turbine modelling

The simplified wind turbine scheme is given in Fig. 1.
From the expression of λ, the aerodynamic power can be rewritten as [8] :

$$P_w = k_T \omega^3 \tag{4}$$

with

$$k_T = \frac{1}{2}\rho\pi \frac{R^5}{\lambda^3} C_p$$

from which the aerodynamic torque is

$$T_w = \frac{P_w}{\omega} = k_T \omega^2 \tag{5}$$

the dynamics of the rotor driven at speed ω are shown to be

$$J_r \dot{\omega} = T_w - B_r \omega - K_r \theta - T_{LS} \tag{6}$$

assuming an ideal gearbox with transmission ratio n, we have

$$n = \frac{T_{LS}}{T_{HS}} = \frac{\omega_g}{\omega} \tag{7}$$

The generator dynamics are given by [3] :

$$J_g \dot{\omega}_g = T_{HS} - B_g \omega_g - K_g \theta_g - T_e \tag{8}$$

and the electromagnetic torque expression of the asynchronous generator is

$$T_e = \frac{m s n_p V^2 R_r}{\omega_{syn}\left[(sR_s + R_r)^2 + (sX)^2\right]} \tag{9}$$

Let us set the state variables as

$$x = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \theta(t) \\ \omega(t) \end{bmatrix}$$

and by letting $u = V^2$, from equations (5)-(8) it yields the following nonlinear state space representation

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{10}$$

with

$$\begin{aligned} f(x) &= \begin{bmatrix} x_2 \\ -\frac{K}{J}x_1 - \frac{B}{J}x_2 + \frac{k_T}{J}{x_2}^2 \end{bmatrix} \\ g(x) &= \begin{bmatrix} 0 \\ -\frac{m s n_p R_r}{J\omega_{syn}[(sR_s+R_r)^2+(sX)^2]} \end{bmatrix} \\ h(x) &= x_2 \end{aligned} \tag{11}$$

IV. NONLINEAR FEEDBACK CONTROL

A. Feedback Linearization Control

It may be shown that the state feedback control law

$$u = \frac{\dot{y}_m - k(y - y_m) - f(x)}{g_2(x)} \tag{12}$$

makes the tracking error dynamics exponentially stable

$$\dot{e} + ke = 0 \tag{13}$$

with $e = y_m - y$ and $k > 0$
hence, the corresponding generator voltage is

$$V = \sqrt{\left|\frac{\dot{y}_m - k(y - y_m) - f(x)}{g_2(x)}\right|} \tag{14}$$

B. Nonlinear Adaptive Control

The major drawback of the input-state linearization based approach presented in the previous section is its lack of robustness under uncertainties. Therefore, we will use parameters adaptive control in order to keep the linearization control law robust with respect to parameters variations.
Let us consider a SISO system of form (10) such that

$$\begin{aligned} f(x) &= \sum_{i=1}^{n1} \theta_i^{(1)} f_i(x) \\ g(x) &= \sum_{j=1}^{n2} \theta_j^{(2)} g_j(x) \end{aligned} \tag{15}$$

where θ_i and θ_j are unknown parameters.
Denote by $\hat{\theta}_i, \hat{\theta}_j$ the estimates of θ_i and θ_j respectively, then, the estimated functions of f and g are

$$\begin{aligned} \hat{f}(x) &= \sum_{i=1}^{n1} \hat{\theta}_i^{(1)} f_j(x) \\ \hat{g}(x) &= \sum_{j=1}^{n2} \hat{\theta}_j^{(2)} g_i(x) \end{aligned} \tag{16}$$

with

$$\begin{aligned} f_1 &= \begin{bmatrix} x_2 \\ 0 \end{bmatrix} \quad ; \quad f_2 = \begin{bmatrix} 0 \\ x_1 \end{bmatrix} \\ f_3 &= \begin{bmatrix} 0 \\ x_2 \end{bmatrix} \quad ; \quad f_4 = \begin{bmatrix} 0 \\ x_2^2 \end{bmatrix} \end{aligned} \tag{17}$$

the nominal values of the parameters are

$$\theta_1 = 1 \quad ; \quad \theta_2 = -\frac{K}{J}$$

$$\theta_3 = -\frac{B}{J} \quad ; \quad \theta_4 = \frac{K_T}{J}$$

From theorems exposed in [9], [10], it is demonstrated that the parameter update law

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u \\ \dot{\hat{\theta}}_2 = e_0 \hat{\theta}_2 x_1 \\ \dot{\hat{\theta}}_3 = e_0 \hat{\theta}_3 x_2 \\ \dot{\hat{\theta}}_4 = e_0 \hat{\theta}_4 {x_2}^2 \\ \dot{\hat{\theta}}_2 = e_0 \hat{\theta}_2 x_1 \\ \dot{\hat{\theta}}_3 = e_0 \hat{\theta}_3 x_2 \\ \dot{\hat{\theta}}_4 = e_0 \hat{\theta}_4 {x_2}^2 \\ \dot{e}_0 = -ke_0 - \tilde{\theta}_2 \hat{\theta}_2 x_1 - \tilde{\theta}_3 \hat{\theta}_3 x_2 \\ \quad -\tilde{\theta}_4 \hat{\theta}_4 {x_2}^2 \end{cases} \tag{18}$$

yields bounded $y(t)$, asymptotically converging to $y_m(t)$ with all the state variables in (10) bounded.
where

$$\tilde{\theta}_i^{(1)} = \hat{\theta}_i^{(1)} - \theta_i^{(1)} \tag{19}$$

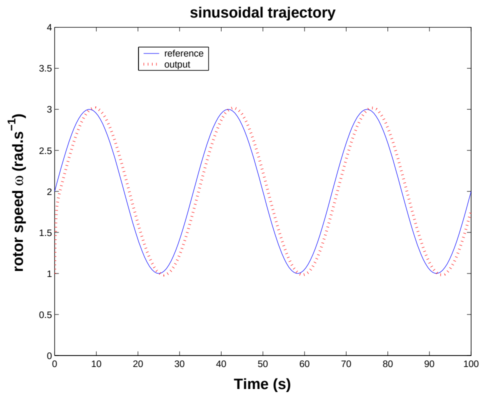
$$\tilde{\theta}_j^{(2)} = \hat{\theta}_j^{(2)} - \theta_j^{(2)} \tag{20}$$

and e_0 an intermediate variable. Finally, the resulting control law is

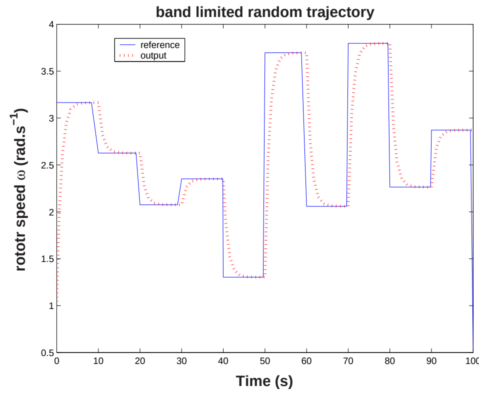
$$\begin{aligned} u = \frac{1}{g_2(x)} \Big[& -(\hat{\theta}_2 x_1 + \hat{\theta}_3 x_2 + \hat{\theta}_4 x_2^2) \\ & + \dot{y}_m + k(y_m - y) \Big] \end{aligned} \tag{21}$$

V. SIMULATION RESULTS

For the proposed nonlinear feedback controllers, both schemes have been applied and simulations have been performed for different rotor speed trajectories.
One may observe that for Feedback Linearization Controller, when the system parameters, used to the synthesis of the control action are equal to the nominal values, the required performance are reached Fig. (2(a)), (2(b)) ; namely the tracking error vanishes for the steady state regime. The time response is about some seconds, there is no overshoot. nevertheless when the wind turbine parameters are

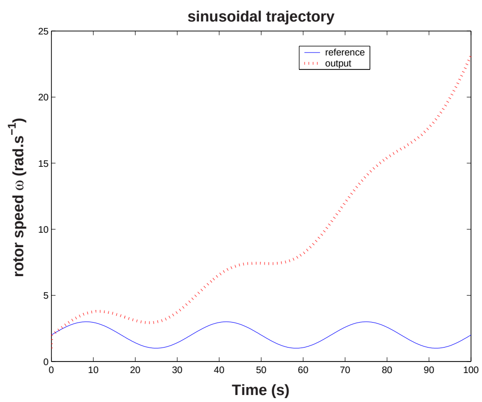


(a) sinusoidal reference

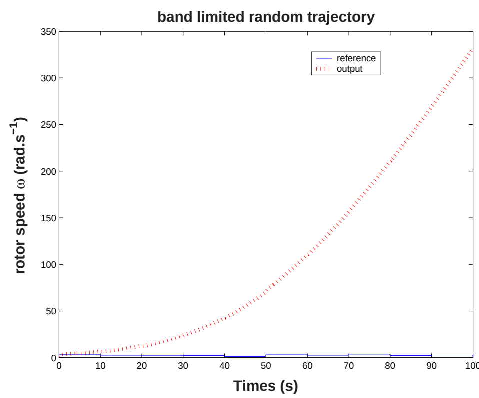


(b) random reference

Fig. 2. Simulation results for the linearizable control law

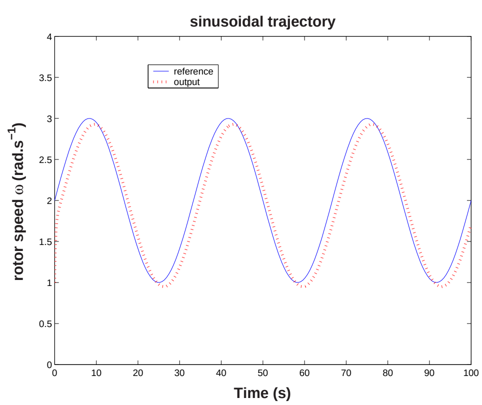


(a) sinusoidal reference

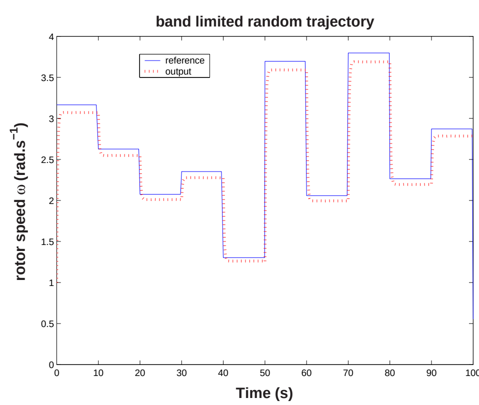


(b) random reference

Fig. 3. Simulation results for the linearizable control law with parameters variations



(a) sinusoidal reference



(b) random reference

Fig. 4. Simulation results for the robust adaptive control law

different from the nominal ones, the tracking performance sensitively decrease as seen in Fig. (3(a)) and (3(b)).

A variation of 40% is carried out in the initial values of the three system parameters $\theta_2,\ \theta_3,$ and θ_4, we keep the initial parameters values in the control law (14).

The use of the Nonlinear Adaptive control law for different trajectories is shown in Fig. (4(a)), (4(b)), the variation of 40% of the system parameters is also used for adaptive controller simulation. One may observe from Fig. (4) a good tracking performance of the rotor speed for different reference trajectories in spite of parameters uncertainties.

VI. Conclusion

This work emphasizes the performance of nonlinear control technics for optimal power curve tracking problem of a wind turbine. The strong nonlinearity of the wind turbine due to the expression of the aerodynamic and electromagnetic torque motivates the use of such technics.

The input-state linearization approach showed not enough robustness level with respect to parameters variations. While nonlinear adaptive controller gives good results in presence of similar uncertainties.

Notation and Symbols

v	mean wind speed $(m.s^{-1})$.
v_{ref}	wind speed reference $(m.s^{-1})$.
ρ	air density $(kg.m^{-3})$.
R	rotor radius (m).
P_w	aerodynamic power (J).
T_w	aerodynamic torque $(N.m)$.
λ	tip speed ratio.
$C_p(\lambda)$	power coefficient.
k_T	power-rotor speed coefficient.
ω	rotor speed $(rad.s^{-1})$.
ω^*	rotor speed reference $(rad.s^{-1})$.
ω_g	generator speed $(rad.s^{-1})$.
θ	rotor angular deviation.
θ_g	rotor angular deviation.
T_{LS}	low speed shaft $(N.m)$.
T_{HS}	high speed shaft $(N.m)$.
T_e	electromagnetic torque $(N.m)$.
n	gearbox ratio.
J_r	rotor inertia $(kg.m^2)$.
J_g	generator inertia $(kg.m^2)$.
J	total inertia $J = J_r + n^2 J_g$ $(kg.m^2)$.
K_r	low speed shaft torsion $(N.rad^{-1})$.
K_g	high speed shaft torsion $(N.rad^{-1})$.
K	total torsion $J = J_r + n^2 J_g$.
B_r	low speed shaft friction $(N.rad^{-1.s})$.
B_g	high speed shaft friction $(N.rad^{-1.s})$.
B	total friction $B = B_r + n^2 B_g$.
ω_{syn}	generator speed $(m.s^{-1})$.
s	the slip $s = (\omega_{syn} - \omega)/\omega_{syn}$.
n_p	number of poles pairs.
m	number of phases.
R_s	rotor resistance (Ω).
R_r	stator resistance (Ω).
X	total inductance (Ω).
V	generator voltage (V).

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