# MULTIOBJECTIVE CONTROL OF A VARIABLE SPEED WIND TURBINE (1)

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***Abstract*— The purpose of this work is to develop a robust controller for variable speed wind turbine based on Multi-objective synthesis in order to optimize the wind power capture in partial load operation (below the rated power) while minimizing the transient loads in the turbine shafts. A linear model of the wind turbine is first deduced from a nonlinear model. Control objectives that associate $H_2$ and $H_\infty$ are formulated in LMI form which is known to offer powerful tools to mixed criterion optimization . The aim of this work is to show that Multi-channel method provide an efficient way to handle a Multi-objective synthesis in variable speed wind turbine control. Simulation results show good performance of the proposed control law for different reference rotor speeds that correspond to proportional wind speed profiles.**

***Index Terms*— wind turbine, variable speed, multiobective control, $H_2/H_\infty$ control**

## I. Introduction

Advances in wind turbine technology [1] necessary imply the design of more powerful control systems. That is to improve wind turbines behavior in order to make them more profitable and more reliable.
The control objective depends in the region where the wind turbine (WT) operates.
Wind turbine operation can be divided into two regions :

- Above rated wind speed (full load) where $v_1 < v < v_2$.
- Below rated wind speed (partial load) where $v_2 < v < v_3$.

$v$ is the mean wind speed.
The wind turbine is stopped for $v < v_1$ and $v > v_3$.
Control system design objectives for each region can be specified by [2] :

- Limitation and smoothing of electrical power in the above rated power area.
- Generation of maximum power in the below rated power area.
- Minimization of transient loads in all turbine components.

Many control strategies have been proposed in the literature, based on LTI models. Classical controllers have been extensively used, particularly the PI regulator [3], [4]. Optimal control of wind turbines have been also used in the $LQ$ [5], [6], and $LQG$ form [2], [7].
Robust control of wind energy conversion systems (WECS) have been introduced in [8] and also used in [9] and [10].
An $H_\infty$ approach using weighting filters for inputs and outputs is presented in [11].

However, as mentioned in [2], the drawback of the methods quoted previously remains in the fact that the control objectives used to controller synthesis are not well formulated to take into account the stochastic and dynamic aspect of wind turbine control.
In the case of variable speed wind turbine (VSWT) control below rated power, the controller must achieve two functions : optimal rotational speed tracking with fast wind speed variations rejection and avoiding significant undergoing efforts (torques and forces) for wind turbine structure.
In [11] those two objectives are treated identically by synthesizing a controller that minimizes the $H_\infty$ norm of the transfer matrix between exogenous inputs (wind speed $v$ and torque disturbance $T_d$) and observed outputs (tracking error and control signal). As known, the $H_\infty$ controller minimizes the worse case of the ratio between the $L_2$ norms of input and output signals.
However in the control problem considered here, it is necessary to minimise the effect of fast wind speed variation over a long horizon while avoiding significant efforts peak to the wind turbine. One has then to use different criterion for each objective.
In this paper, a multiobjective $H_\infty/H_2$ control is used to build robust controllers for a horizontal axis variable speed wind turbine.
This paper is organized as follows. Section II presents the system formulation and the linearized model of the wind turbine. Section III describes the problem formulation and the multiobjective control approach using LMI to achieve a robust controller that takes into account different design specifications. In section IV, several simulation results illustrate the performance of the proposed approach.

## II. Wind turbine modelling

### A. Model description

The wind turbine model is composed of the rotor aerodynamics, the gearbox and an asynchronous generator. The

(1) This work has been carried out within the project €nergie launched by Supélec under industrial support.

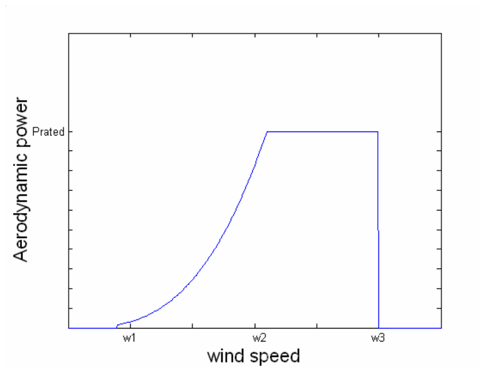


Fig. 1. Wind turbine power curve

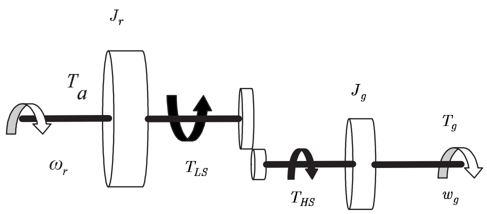


Fig. 2. Wind turbine scheme

simplified wind turbine scheme is given in Fig. 2
The aerodynamic power expression is given by

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda)v^3 \quad (1)$$

Using the relation ship

$$P_a = \omega_r T_a \quad (2)$$

aerodynamic torque expression is then

$$T_a = \frac{1}{2}\rho\pi R^3 C_q(\lambda)v^2 \quad (3)$$

with

$$C_q(\lambda) = \frac{C_p(\lambda)}{\lambda}$$

the dynamics of the rotor driven at speed $\omega_r$ are shown to be

$$J_r\dot{\omega}_r = T_a - T_{LS} \quad (4)$$

$T_{LS}$ is the low speed shaft whose expression is

$$T_{LS} = B_s(\omega_r - \omega_{LS}) - K_s(\theta_r - \theta_{LS}) \quad (5)$$

assuming an ideal gearbox with transmission ratio $n$ we have

$$n = \frac{T_{LS}}{T_{HS}} = \frac{\omega_g}{\omega_{LS}} = \frac{\theta_g}{\theta_{LS}} \quad (6)$$

The generator dynamics are given by

$$J_g\dot{\omega}_g = T_{HS} - T_g \quad (7)$$

where $T_g$ is the electromagnetic torque.
The wind turbine model has two inputs : an uncontrollable input that is the wind speed $v$ and the generator torque that constitutes the control input.
The choice of $T_g$ as a control input is motivated by the fact that when connecting the generator to the grid via a frequency converter, the generator rotational speed $\omega_g$ will be independent of the grid frequency. By controlling the firing angle of the converter it is possible to control the electrical torque in the generator. The torque control using the frequency converter allows the wind turbine to run at variable speed and thereby make possible a reduction of the stress on the drive train and gearbox [3].

### B. Linearized model

The nonlinearity of the model described in the previous section is contained in the aerodynamic torque expression (3). The linearization of the aerodynamic torque, around an operating point leads to the following relation ship

$$\Delta T_a = \alpha\Delta v + \eta\Delta\omega_r \quad (8)$$

where

$$\begin{cases} \alpha = \dfrac{\partial T_a(v_0,\omega_0)}{\partial v} = \dfrac{1}{2}\pi\rho R^3 v_0\left[2C_q(\lambda_0) - \lambda_0 C_q'(\lambda_0)\right] \\ \eta = \dfrac{\partial T_a(v_0,\omega_0)}{\partial \omega_r} = \dfrac{1}{2}\pi\rho R^3 v_0 R C_q'(\lambda_0) \end{cases} \quad (9)$$

by replacing from (6) $\omega_{LS} = \frac{\omega_g}{n}$ and $\theta_{LS} = \frac{\theta_g}{n}$ in (5), and $T_{HS} = \frac{T_{LS}}{n}$ in (7), one can write down

$$\begin{aligned} T_{LS} &= B_s(\omega_r - \frac{\omega_g}{n}) - K_s(\theta_r - \theta_{LS}) \\ \dot{\omega}_g &= \frac{T_{LS}}{nJ_g} - \frac{T_g}{J_g} \end{aligned} \quad (10)$$

Let us now

$$\theta_d = \theta_r - \theta_{LS} = \theta_r - \frac{\theta_g}{n}$$

By using (4) and (8)-(10), and while taking as state variables the variation of the considered quantities around their steady state values, we obtain the following linear state space representation

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (11)$$

with :

$$x = [\theta_d \quad \omega_r \quad \omega_g]^T \quad : \text{ state vector}$$

$$u = [v \quad T_g]^T \quad : \text{ input vector}$$

$$A = \begin{bmatrix} 0 & 1 & -\frac{1}{n} \\ -\frac{K_s}{J_r} & \frac{\eta - C_s}{J_r} & \frac{C_s}{nJ_r} \\ \frac{K_s}{nJ_g} & \frac{C_s}{nJ_g} & -\left(\frac{C_s}{n^2 J_g}\right) \end{bmatrix} \quad : \text{ state matrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{\alpha}{J_r} & 0 \\ 0 & -\frac{1}{J_g} \end{bmatrix} \quad : \text{ input matrix}$$

## III. Mutiobjective control

### A. problem formulation

The wind speed time repartition makes that, in general, the wind turbines operating in wind speed less than rated one, hence the importance of control efficiency arises in this operating regime. While energy is captured from the wind, the

aerodynamic power should be maximized.
The $C_p(\lambda)$ curve which appears in the aerodynamic power expression (1) has a unique maximum at

$$\lambda_{opt} = \frac{\omega_r^* R}{v} \tag{12}$$

that corresponds to a maximum power production.
In order to make $\lambda$ tracking its optimal value, the rotor speed is then adjusted to track the reference $\omega_r^*$ which has the same shape as wind speed since they are proportional.

$$\omega_r^* = \frac{\lambda_{opt}}{R} v \tag{13}$$

As already mentioned, the objective of the controller is to maximize wind power extraction by adjusting the rotor rotational speed $\omega$ to wind speed variation such that the aerodynamic power stands at its maximum in spite of this variations.
The controller objectives are then

1) Minimizing the effect of wind speed fast variations
2) Reducing the stress undergoes by the wind turbine parts
3) Tracking the optimal rotor speed $\omega_r^*$

The first objective can be achieved using an $H_2$ criterion that corresponds to the minimization of a disturbance effect over a long horizon. While the second is equivalent to avoid to the wind turbine significant effort peak, thus the worst case. This can be reached using an $H_\infty$ synthesis.

### *B. $H_2/H_\infty$ control*

From the diagram of Fig. 3, the $H_2$ $(resp.\ H_\infty)$ controller synthesis problem can be formulated as finding a controller $K$ over the set of all stabilizing controllers that minimizes the $H_2$ $(resp.\ H_\infty)$ norm of the Linear Fractional Transformation (LFT) $T_{zw}$

$$T_{zw} = G_{zu}(s)K(s)\left(I - G_{yu}(s)K(s)\right)^{-1} G_{yw}(s) + G_{zw}$$

and where the $H_2$ $(resp.\ H_\infty)$ norms of transfer matrix $T$ are

$$\begin{aligned} \|T\|_2 &= \left(\frac{1}{2}\int_{-\infty}^{\infty} trace[T^*(j\omega)T(j\omega)]\, d\omega\right)^{1/2} \\ \|T\|_\infty &= \sup_{\omega}\ \sigma_{max}\left[G(j\omega)\right] \end{aligned}$$

The problem solved in state space approach [12] give a systematic approach for the synthesis of an optimal $H_2$ or $H_\infty$ controller using DGKF (Doyle, Glover, Khagounekar and Francis) algorithm [13], [12].
However, those two standard approaches, used independently, are not adequate with the whole design specifications. For instance, noise attenuation or regulation against random disturbances are more naturally expressed in $LQG$ terms. Similarly, pure $H_\infty$ synthesis only enforces closed-loop stability and does not allow for direct placement of the closed-loop poles in more specific regions of the left-half plane [14].
The Multi-objective design procedures simultaneously take into account several performance criterion, the principle of these methods is to define several channels associated to different criterions.
Mixed $H_2/H_\infty$ is used in this work to reach design specifications as defined in the previous subsection (Fig. 3).

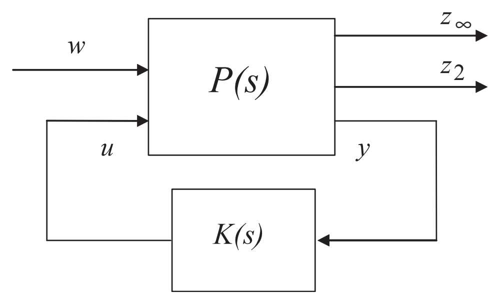


Fig. 3. $H_2/H_\infty$ synthesis

We therefore define then two output channels associated with both $H_2$ and $H_\infty$ criterions.
The output channel $z_\infty$ is associated to the $H_\infty$ performance while the channel $z_2$ is associated to the $LQG$ aspects ($H_2$ performance in the case of white noise disturbance).
The problem became a multiobjective optimization problem with three criterions to minimize :

$$\begin{array}{lcl} \text{Criterion 1} & : & \|T\|_2 \\ \text{Criterion 2} & : & \|T\|_\infty \\ \text{Criterion 3} & : & a\|T\|_\infty + b\|T\|_2 \end{array} \tag{14}$$

The third criterion is a trade-off between the two first.
LMI formulation is used to solve this problem for the considered case.

### *C. LMI formulation*

State space equation (11) may be rewritten as

$$\left\{ \begin{array}{lcl} \dot{x} &=& Ax + B_1 w + B_2 T_g \\ z_\infty &=& C_\infty X + D_{\infty 1} w + D_{\infty 2} T_g \\ z_2 &=& C_2 + D_{21} w + D_{22} T_g \\ y &=& C_y X + D_{y1} w \end{array} \right. \tag{15}$$

or equivalently

$$\begin{bmatrix} \dot{x} \\ \hline z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u \end{bmatrix} \tag{16}$$

with

$$\begin{aligned} z &= \begin{bmatrix} z_\infty \\ z_2 \end{bmatrix} = \begin{bmatrix} T_g \\ \omega_g \end{bmatrix} \quad ; \quad u = T_c \\ w &= \begin{bmatrix} T_d \\ v \end{bmatrix} \quad ; \quad y = \omega_g \\ T_g &= T_d + T_c \end{aligned}$$

and

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{\alpha}{J_r} \\ 0 & -\frac{1}{J_r} \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J_r} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad D_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad D_{22} = 0$$

$T_c$ and $T_d$ are control and disturbance torque respectively.
The $z_2$ channel $\omega_g$ is associated to the reduction of the torque disturbance effect and fast wind speed variations on the optimal generator speed $\omega_g^*$,

$$\omega_g^* = n\omega_r^* \tag{17}$$

$\omega_r$ is defined in (13). It corresponds to $H_2$ performance.
While the $T_g$ channel is associated to the $H_\infty$ performance that corresponds to the reduction of the stress undergone by the wind turbine.
From the generalized state space representation (16), it is shown in [15] that one can construct under certain conditions, via LMI optimization, a robust controller which is a solution of the multi-criterion problem (14).
The solution is a controller $K$ under a state representation

$$\begin{cases} \dot{\xi} = A_K\xi + B_K u \\ y = C_K\xi + D_k u \end{cases} \tag{18}$$

Details for controller synthesis are given in [15].

## IV. Simulation Results

The wind turbine considered in this study is a variable speed WT. The variable speed option interest comes out from the fact that it reduces stress due to the transient loads in the main shaft during the full load operation of the wind turbine and optimizes energy extraction over all wind speeds below rated. An additional benefit is that the variable speed turbines rotate far less during their life time; i.e. they can be brought to a lower rotational speed in the low wind speed region.
The proposed multiobjective $H_2/H_\infty$ controllers, both schemes have been applied and simulations have been performed for different rotor speeds.
The obtained results are shown in Fig. 4 - 7.
One may observe that for $H_\infty$ controller, the tracking error for a noisy wind speed step is large enough Fig. 5. The steady state is not reached even after 10 s. The Bode diagram of this controller is shown in Fig. 4, and it's transfer function is

$$K_\infty = \frac{0.074985(s - 6.692e009)(s^2 + 15.43s + 347.9)}{(s + 3.753e004)(s + 1.243e004)(s + 108.9)} \tag{19}$$

The use of Mixte $H_2/H_\infty$ criterions for the synthesis of robust controller is shown in Fig. 6, 7. One may observe from Fig. 7 a good tracking performance of the generator speed in spite of fast wind speed variations.

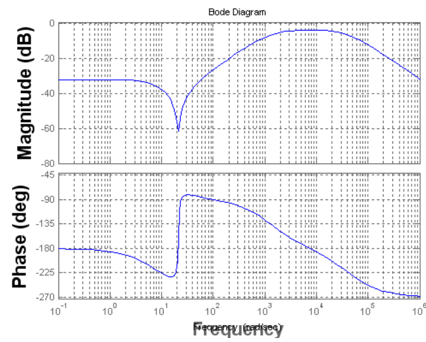


Fig. 4. $H_\infty$ controller Bode Diagram

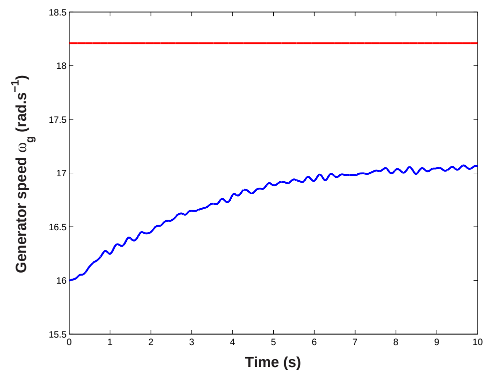


Fig. 5. Closed Loop system response to a step of noisy wind with the $H_\infty$ controller

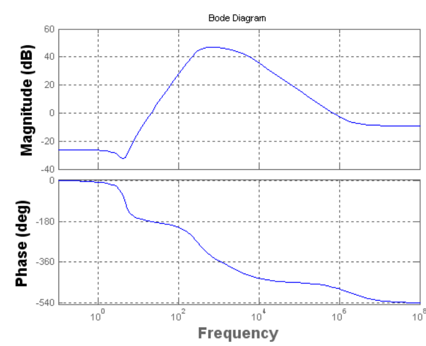


Fig. 6. $H_2/H_\infty$ controller Bode Diagram

The robustness of the controller leads to good performance despite the noise and the input step magnitude signal.
The time response is about one second, and the overshot is small.
The transfer function of the $H_2/H_\infty$ controller is found to be

$$K_{H_2/H_\infty} = \frac{-25509.7484(s^2 + 1.854s + 450.9)}{(s + 3.917e004)(s + 1335)(s + 8.992)} \tag{20}$$

and the corresponding Bode diagram is given in Fig. 6.
The $H_2/H_\infty$ controller is also tested for a sinusoidal disturbed wind input that corresponds to a sinusoidal generator speed reference.
Similarely, one may observe from Fig. 8 a good tracking performance of the generator speed for the sinusoidal reference trajectory in spite of wind disturbance.

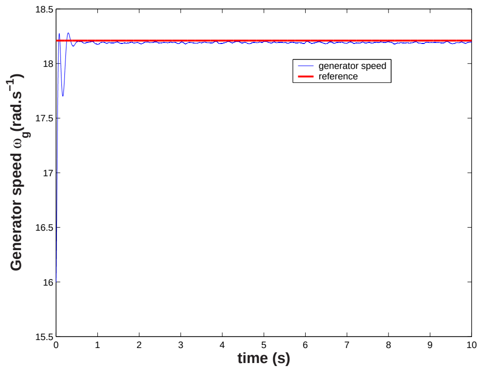


Fig. 7. Closed Loop system response to a step of noisy wind with the $H_2/H_\infty$ controller

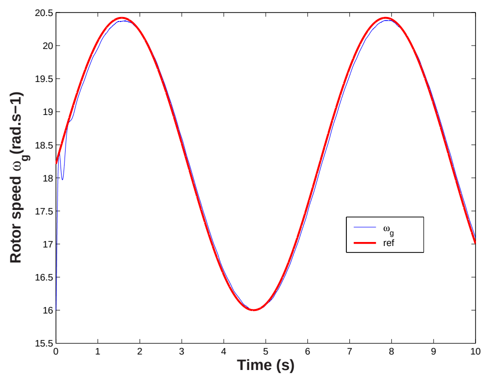


Fig. 8. Closed Loop system response to a sinusoidal noisy wind with the $H_2/H_\infty$ controller

## V. Conclusion

This work emphasizes the performance of multiobjective $H_2/H_\infty$ control technics for optimal power curve tracking problem of a wind turbine. This multi-criterion permitted the synthesis of robust controller satisfying two objectives that cannot being formulated with a same performance criterion. Those robust controllers showed a good robustness with respect to classical ones and gave satisfactory results in presence of input disturbances.

## Appendix I
## Notation and Symbols

| | |
|---|---|
| $v$ | mean wind speed $(m.s^{-1})$. |
| $v_{ref}$ | wind speed reference $(m.s^{-1})$. |
| $\rho$ | air density $(kg.m^{-3})$. |
| $R$ | rotor radius $(m)$. |
| $P_a$ | aerodynamic power $(W)$. |
| $T_a$ | aerodynamic torque $(N.m)$. |
| $\lambda$ | tip speed ratio. |
| $C_p(\lambda)$ | power coefficient. |
| $C_q(\lambda)$ | torque coefficient. |
| $\omega_r$ | rotor speed $(rad.s^{-1})$. |
| $\omega^*$ | rotor speed reference $(rad.s^{-1})$. |
| $\omega_g$ | generator speed $(rad.s^{-1})$. |
| $\theta_d$ | rotor angular deviation $(\theta \in [0, 2\pi))$. |
| $\theta_g$ | rotor angular deviation $(\theta_g \in [0, 2\pi))$. |
| $T_{LS}$ | low speed shaft $(N.m)$. |
| $T_{HS}$ | high speed shaft $(N.m)$. |
| $T_g$ | generator (electromagnetic) torque $(N.m)$. |
| $n$ | gearbox ratio. |
| $J_r$ | rotor inertia $(kg.m^2)$. |
| $J_g$ | generator inertia $(kg.m^2)$. |
| $K_S$ | low speed shaft torsion $(N.rad^{-1})$. |
| $C_s$ | low speed shaft friction $(N.rad^{-1}.s)$. |

## Acknowledgment

The authors would like to thank ABBAS-TURKI Mohamed for very helpfull discussions.

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