

# Model Predictive Control of a Variable Speed Wind Turbine Using A Two-mass Model

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**Abstract**—A model predictive controller is proposed for a variable speed wind turbine. The controller is designed using a two-mass model of the aeroturbine. This allows dealing with the drive train stiffness and damping. The controller is synthesized to maximize wind power capture while reducing stress on the mechanical parts of the wind turbine. The model predictive controller performances are tested with the parameters of a real experimental wind turbine under realistic wind speed conditions. It achieves an acceptable performance.

**Index Terms**—Wind turbine, Predictive control, Renewable energy

## I. INTRODUCTION

Since many decades, wind energy systems have received growing interest [1]. This is due to several reasons: Wind energy is clean. It is also a renewable source of energy. Add to this, it is a resource that is available in most regions of the planet [2].

Control design is a key factor for improving wind turbines efficiency [3]. Many control strategies are applied to achieve better performances [4]. Predictive control is a well known control technique. It has a lot of success particularly in industrial applications such as chemical process [5]. A review of predictive control of wind turbines is given in [6]. In the major part of the works dealing with predictive control of a wind turbine, the authors consider a detailed model of the wind turbine generator. Two-phase models are often used for DFIG or permanent magnet generators. The control objective is to achieve an independent control of active and reactive power produced by the wind turbine [7]. However, this works consider a simplified model of the aeroturbine and the wind. Frequently, a constant step wind speed is used to test the controllers. In other cases, a variable wind speed profile is used. Those profiles are generally short and unrealistic. It is important to test the controller with a realistic and sufficiently long wind speed profile. In this work, a more detailed model of the aeroturbine is considered. The chosen model is flexible. It describes the drive train by considering a flexible aeroturbine shaft. Add to this, a realistic wind speed profile with a sufficient duration is used. The model predictive controller is designed in the aim of improving wind power capture while reducing stress on the aeroturbine shaft.

This paper is organized as follows: The two-mass model used for control design is detailed in section 2. Section 3

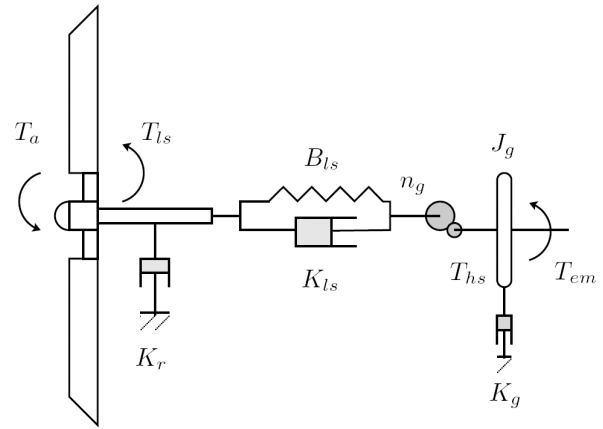


Fig. 1. Two-mass wind turbine.

summarizes the control objectives for a variable speed wind turbine while focusing on the low wind speed region. The proposed model predictive controller is detailed in section 4. The controller is tested with a realistic wind turbine parameters and wind speed profile. The simulation results are analyzed and commented. A conclusion and perspectives are given at the end of the paper.

## II. TWO-MASS WIND TURBINE MODEL

A wind turbine is a complex mechatronic system. It converts a part of the wind power to electrical power. A wide variety of models are described in the literature. The model structure is related to its use. A two-mass model is an acceptable compromise between simplicity and relevancy. It is a reduced complexity model. However, it can highlight important phenomena such as stress and vibrations of the drive train. The used two-mass model is depicted in Fig. 1. It consists of two rotating inertias related with a flexible shaft and a gearbox. The mathematical equations that describe the two-mass behavior are given by Eq. (1)

$$\begin{cases} J_r \dot{\omega}_t &= T_a - K_r \omega_t - T_{ls} \\ J_g \dot{\omega}_g &= T_{hs} - K_g \omega_g - T_{em} \\ T_{ls} &= K_{ls} (\omega_t - \frac{\omega_g}{n_g}) + B_{ls} (\theta_t - \frac{\theta_g}{n_g}) \end{cases} \quad (1)$$

The values and description of the symbols in equation (1) are given in table III.

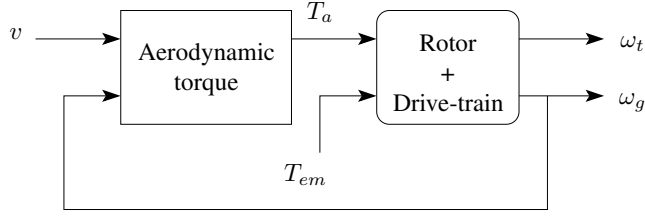


Fig. 2. Wind turbine block diagram.

The ratio between the low-speed and the high-speed variables is  $n_g$  as shown in the following equation

$$n_g = \frac{T_{ls}}{T_{hs}} = \frac{\omega_g}{\omega_t} = \frac{\theta_g}{\theta_t}, \quad (2)$$

Combining equations (1) and (2), a state-space representation of the wind turbine is deduced

$$\begin{bmatrix} \dot{\omega}_t \\ \dot{\omega}_g \\ \dot{T}_{ls} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \omega_t \\ \omega_g \\ T_{ls} \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} T_a + \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} T_{em} \quad (3)$$

with

$$\begin{aligned} a_{11} &= -\frac{K_r}{J_r} ; & a_{12} &= 0 ; & a_{13} &= -\frac{1}{J_r} \\ a_{21} &= 0 ; & a_{22} &= -\frac{K_g}{J_g} ; & a_{23} &= \frac{1}{n_g J_g} \\ a_{31} &= \left( B_{ls} - \frac{K_{ls} K_r}{J_r} \right) ; & a_{32} &= \frac{1}{n_g} \left( \frac{K_{ls} K_r}{J_g} - B_{ls} \right) \\ a_{33} &= -K_{ls} \left( \frac{J_r + n_g^2 J_g}{n_g^2 J_g J_r} \right) \end{aligned}$$

and

$$\begin{aligned} b_{11} &= \frac{1}{J_r} ; & b_{12} &= 0 \\ b_{21} &= 0 ; & b_{22} &= -\frac{1}{J_g} \\ b_{31} &= \frac{K_{ls}}{J_r} ; & b_{32} &= \frac{K_{ls}}{n_g J_g} \end{aligned}$$

As shown in Fig. 2, the model can be seen as linear with two inputs : The electromagnetic torque  $T_{em}$  that is a control input and the aerodynamic torque  $T_a$  that is a disturbance input. Instead of its simplicity, the proposed approach should be compared in future works with nonlinear predictive control methods where the aerodynamic torque is not considered as a disturbance input but as a nonlinear function of the wind and the turbine variables.

### III. CONTROL OBJECTIVES

The control objective of a wind turbine depends on its operating region. As depicted in Fig. 3, there are basically three operating regions depending on the wind speed :

**Region 1** The wind turbine is stopped. The wind speed  $v$  is too low to start the wind turbine,

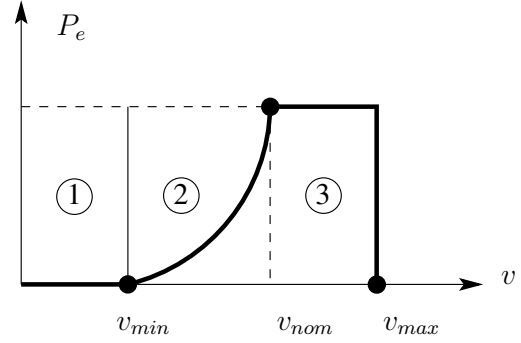


Fig. 3. Wind turbine operation.

**Region 2** The wind turbine is controlled to rotate at an optimal speed. This speed allows capturing the maximum power from the wind,

**Region 3** Either the wind turbine rotational speed and produced power are regulated to their nominal values.

In this work, only the region 2 shown in Fig. 3 is considered. It is associated with the low wind speed values. More precisely, the controller in this region should achieve :

- Tracking the optimal aerodynamic power,
- Reduce stress on wind turbine components,
- Apply smooth and acceptable control actions.

for a given wind speed  $v$ , the optimal captured aerodynamic power is

$$P_{a_{opt}} = \frac{1}{2} \rho \pi R^2 C_{p_{opt}} v^3, \quad (4)$$

In order to track the optimal aerodynamic power, the power coefficient  $C_p(\lambda, \beta)$  should be maintained at its maximum value  $C_{p_{opt}}$ . This one corresponds to optimal values of the tip speed ratio  $\lambda_{opt}$  and blade pitch angle  $\beta_{opt}$

$$C_{p_{opt}} = C_p(\lambda_{opt}, \beta_{opt}) \quad (5)$$

For this, the blade pitch angle is fixed to  $\beta_{opt}$  and the tip speed ratio to  $\lambda_{opt}$ . The wind turbine rotor has then to track an optimal reference speed

$$\omega_{t_{opt}} = \frac{\lambda_{opt}}{R} v \quad (6)$$

One can see that the optimal rotor speed  $\omega_{t_{opt}}$  is proportional to the wind speed  $v$ . Conversely, the wind speed can be modeled as the sum of constant mean value and a turbulent component. For this, a close tracking of the instantaneous optimal rotor speed will lead to a turbulent behavior of the rotor speed. It causes high stress in the wind turbine components, especially in the drive train and in the rotor shaft. For this, a compromise should be considered between power capture optimization and stress reducing. The adopted approach using a model based predictive controller is detailed in the next sections.

#### IV. MODEL PREDICTIVE CONTROLLER

Many control methods are described in the literature to design a predictive controller. Nevertheless, all these methods share the same principle for predictive control : use a model to predict the future outputs of the system as a function of current and future control inputs, and then, optimize these ones against a given criterion. The choice of a given predictive technique is guided by the problem specificity.

For the problem considered in this work, the two-mass model presented in section 2 can be seen as a linear multivariable system with two inputs: The aerodynamic torque  $T_a$  seen as a disturbance input and the electromagnetic torque  $T_{em}$  that is the control input. The aerodynamic torque may be considered as a measurable disturbance. In fact, using an anemometer or a LIDAR system allows measuring the wind speed  $v$  [8]. The aerodynamic torque value  $T_a$  can then be deduced if the wind turbine characteristics are well known. If the wind speed is not available, an estimator, as a Kalman filter, can be used to deduce the aerodynamic torque estimate  $\hat{T}_a$  [9]. As a measured output, either the rotor speed  $\omega_t$  or the generator speed  $\omega_g$  is considered. In fact, they are almost proportional by the ratio  $n_g$  of the drive train

$$\omega_g \approx n_g \omega_t \quad (7)$$

In the flowing text, the rotor speed is considered as the measured output. If a speed sensor is only available on the generator, it is then deduce as

$$\omega_t \approx \frac{1}{n_g} \omega_g \quad (8)$$

Considering a prediction horizon  $n_p$ , the future system outputs deduced using the two-mass wind turbine model are

$$\hat{Y} = [ \omega_t(k+1) \quad \omega_t(k+2) \quad \omega_t(k+n_p) ] \quad (9)$$

The current and future control sequence is

$$U(k) = [ T_{em}(k) \quad \dots \quad T_{em}(k+n_u-1) ]^T \quad (10)$$

where  $n_u$  is the control horizon. This control sequence is calculated to optimize the following criterion

$$J(U(k)) = J_{\omega_t}(U(k)) + J_{T_{em}}(U(k)) + J_{\Delta T_{em}}(U(k)) + J_{\varepsilon}(U(k)) \quad (11)$$

with

$$J_{\omega_t}(U(k)) = \sum_{i=1}^{n_p} w_i^y (\omega_{t_{opt}}(k+i) - \omega_t(k+i))^2 \quad (12)$$

$w_i^y$  is the weighting of the  $i$ -th prediction error. The reference  $\omega_{t_{opt}}$  is given by equation (6).

$$J_{T_{em}}(U(k)) = \sum_{i=0}^{n_u-1} w_i^u (T_{em}(k+i) - T_{em_{ref}}(k+i)) \quad (13)$$

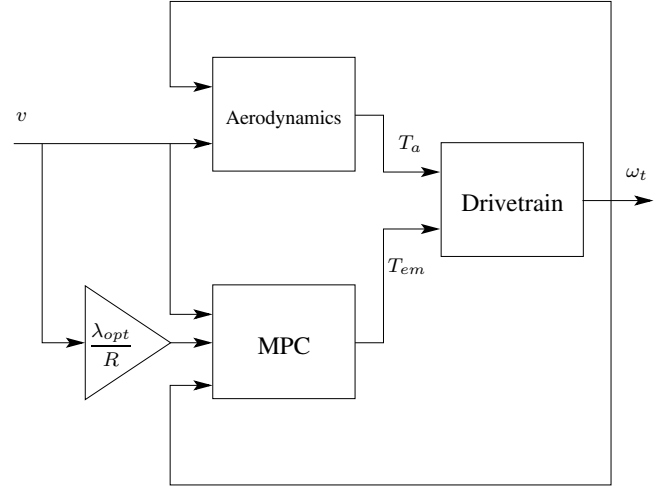


Fig. 4. MPC controller bloc diagram.

$T_{em_{ref}}(k+i)$  is the target control signal at instant  $(k+i)$  and  $w_i^u$  is the weighting of the  $i$ -th control error

$$J_{\Delta T_{em}}(U(k)) = \sum_{i=0}^{n_u-1} w_i^{\Delta u} (T_{em}(k+i) - T_{em_{ref}}(k+i-1))^2 \quad (14)$$

and finally

$$J_{\varepsilon}(U(k)) = \rho_{\varepsilon} \varepsilon_k \quad (15)$$

with  $\rho_{\varepsilon}$  a slack variable at instant  $k$  to penalize constraint violation at this instant.  $\varepsilon_k$  is the slack variable weight.

The optimization of criterion  $J(U(k))$  is a quadratic programming problem under constraints. The Matlab Model Predictive Control Toolbox is used to solve this problem using a solver implemented in a Matlab solver function called quadprog. The solver is included in Matlab optimization toolbox. The solver is based on the interior-point-convex algorithm [10].

An important idea in this work is to consider the aerodynamic torque  $T_a$  as a measurable disturbance and to compensate it. The aeroturbine model is then linear. The quadratic programming techniques are in this case able to reach an optimum of the performance criterion  $J(U(k))$ . The bloc diagram of the proposed MPC controlled is illustrated in Fig. 4. The aerodynamic model is used to predict the disturbance  $T_a$ . It is assumed that the wind speed is measurable.

#### V. SIMULATION RESULTS

The proposed predictive controller is tested in simulation. The used wind turbine is an experimental two-bladed wind turbine named CART. CART (Controls Advanced Research Turbine) is situated in the NREL<sup>1</sup>. It is a pitch controlled wind turbine equipped with an asynchronous generator and power converters to control independently the electromagnetic torque  $T_{em}$  acting on the generator. CART is a medium scale wind

<sup>1</sup>National Renewable Energy Laboratory, Golden, Co., USA

TABLE I  
CART PARAMETERS [11].

Parameter	Value
Blade rotor radius $R$	21.65 m
Gearbox ratio $n_g$	43.165
Hub height	36.3 m
Generator Nominal power	600 kW

TABLE II  
MPC CONTROLLER PARAMETERS.

Parameter	Value
Prediction horizon $n_p$	10
Control horizon $n_u, n_g$	2
Output tracking error weight $w^y$	1.e3
Control tracking error weight	1

turbine of 600kW nominal power. The CART parameters are given in Table I

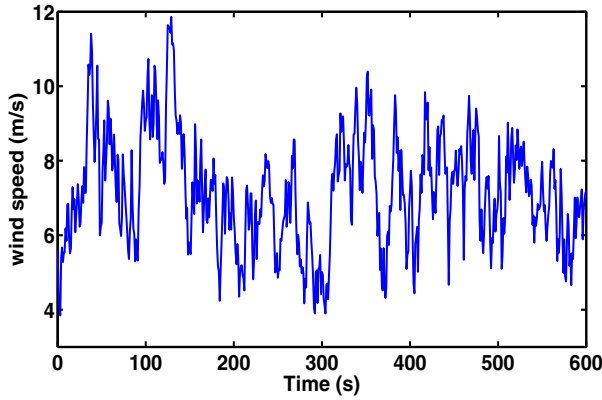


Fig. 5. Wind speed profile of  $7 \text{ m} \cdot \text{s}^{-1}$  mean value.

#### A. Simulation conditions and results

A realistic wind speed profile is used to test the controller. It has a mean value of 7 m/s and a turbulence of 25%.

The wind speed profile is generated using SNwind [12]. The wind speed profile is depicted in Fig. 5

The model predictive controller parameters are summarized in Table 2. The controller is designed using Model Predictive Control Toolbox under MATLAB 2008B. The simulation results are presented in Fig. 6, 7 and 8. It should be noticed that the designed controller is a partial one as it only deals with low wind speed area. A full controller that considers all the wind speeds as shown in section III in future works.

#### B. Results analysis

As shown in Fig. 6, the rotor speed tracks the mean tendency of the optimal rotor speed. This allows a good compromise between power capture optimization and stress reduction. The generator torque is shown in Fig. 7. Either than the rotor speed tracks the mean tendency of the optimal rotor speed,

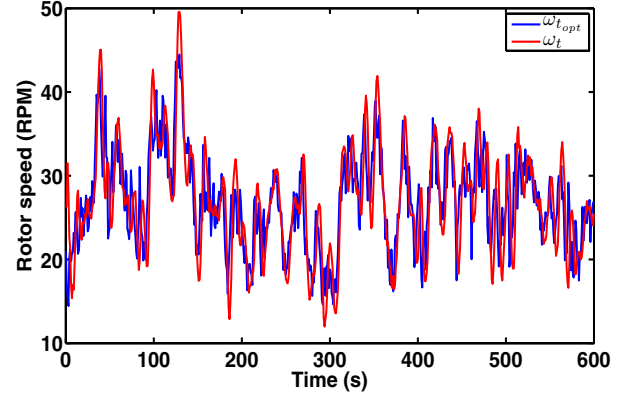


Fig. 6. Rotor speed.

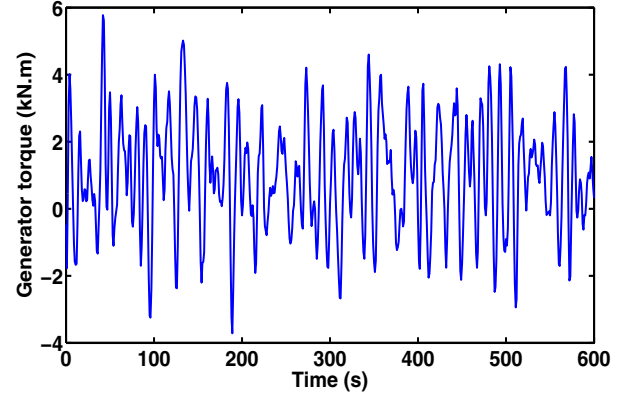


Fig. 7. Generator torque.

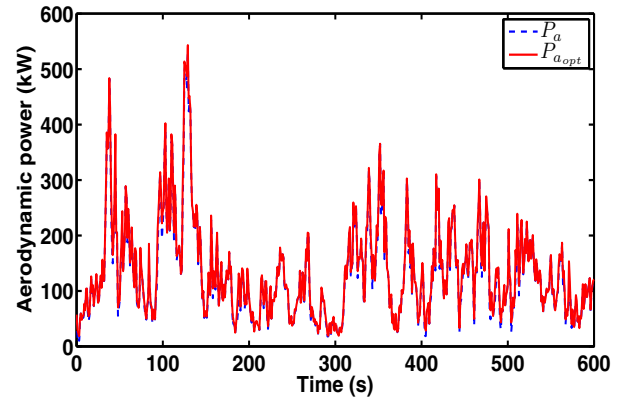


Fig. 8. Aerodynamic power.

the aerodynamic power, as shown in Fig. 8 is very close to the optimal value. Robustness and disturbance rejection are not considered in this work. They will be in future ones.

## VI. CONCLUSION

A simplified model predictive controller is proposed for a variable speed wind turbine. It is based on a two-mass

model of a variable speed wind turbine. The main idea behind the controller is to consider the aerodynamic torque as a measurable disturbance. This allows considering the wind turbine model as linear. Quadratic programming algorithms convergence to a local minimum is then guaranteed. A linear predictive controller is then deduced. The controller shows good performances in terms of power capture optimization. The same technique should be extended in future works for the whole operating area of the wind turbine including high-wind speeds.

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#### APPENDIX

TABLE III  
NUMERICAL VALUES AND SIGNIFICANCE OF CART WT PARAMETERS

Parameter	Value
Rotor radius	$R = 21.65 \text{ m}$
Air density	$\rho = 1.29 \text{ kg/m}^3$
Rotor inertia	$J_r = 3.25 \cdot 10^5 \text{ kg.m}^2$
Generator inertia	$J_g = 34.4 \text{ kg.m}^2$
Shaft damping	$K_{ls} = 9500 \text{ N.m/rad/s}$
Shaft stiffness	$B_{ls} = 2.691 \cdot 10^5 \text{ N.m/rad}$
Rotor damping	$K_r = 27.36 \text{ N.m/rad/s}$
Generator damping	$K_g = 0.2 \text{ N.m/rad/s}$
Gearbox ratio	$n_g = 43.165$

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