Gain Scheduling: A Short Review

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***Abstract*—In this paper, a review about gain scheduling (GS) is presented, with a special focus on the classical GS approach and the linear parameter varying (LPV) approach. The main definitions are given. A brief overview of stability and performance is stated. More recent results are gathered. Finally, a simulation example on a rotational link is presented, comparing two different techniques of gain scheduling, showing that gain-scheduled LPV state feedback actually achieves better performance over a much wide operating range than classical gain scheduling.**

***Index Terms*—Gain scheduling (GS) control, Linear parameter-varying systems (LPV), Linearization**

I. INTRODUCTION

Last decades have seen the rapid development of the systematic design of controllers that can guarantee both stability and performance for nonlinear systems [1]. The research emphasis has moved from focusing solely on optimality to also taking into account robustness in the presence of parameter variations. Gain-scheduling can be applied if these parameters are available online for measurement. The key idea behind gain-scheduling is to decompose the nonlinear problem into linear or nonlinear sub-problems, the main difference lies in the implementation [2]. It is widely known that this controller design approach provides better stability and performance results for slowly varying parameters [3]. The Linear parameter varying approach firstly introduced by Shamma is systematic way of performing gain-scheduled controllers [4]. The passed twenty years have seen increasingly rapid advances on gain scheduling in both practical and theoretical results, we cite for example the work presented in [5] and [6]. As a result it becomes widely used in many engineering applications such as missile autopilte [7],autonomous vehicules [8], Autonomous Aerial Vehicles [9] and wind turbines [10], [11]. Many approaches to gain scheduling can be found in the literature we can classify them in many different ways, according to the existed main methods: classical gain scheduling, Linear Parameter Varying (LPV) gain scheduling, Linear Fractional Transformation (LFT) based gain scheduling, Fuzzy gain scheduling, Model predictive control (MPC) gain scheduling, and other new methods. According to the nature of the signal, it is possible to find continuous or discrete, hybrid or switched gain scheduling controllers. It depends also on the decomposition of the nonlinear design into linear or affine nonlinear sub-problems [12].

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In this work, we attempt to briefly review the main theoretical results on gain scheduling (GS) with a special focus on classical GS and LPV based GS.

This paper is devided into six sections. Section II gives the main definitions. Section III presents the classical gain scheduling. Section IV is devoted to LPV gain scheduling. In section V a simulation example on a simple rotational link is made using classical and LPV approach. Finally, the last section contain conclusions and future work.

Throughout this paper, * will refer to Matrix entries that can be inferred by symmetry.

II. PRELIMINARIES

In this section main definitions are given

Definition 1: **Scheduling Variable** A variable that represents the dynamic changes of the plant and is a measurable signal.

Definition 2: **Scheduled Variable** A variable that changes as a function of a scheduling variable, as a result of that changing the controller will change according to the operating point.

Definition 3: **Exogenous parameters** are external variables. The system is in that case non stationary.

Definition 4: **Endogenous parameters** are function of the state variables,$\rho(x(t), t)$, In this case, the LPV system is called a quasi-LPV system. This is the case when approaching nonlinear systems [13].

III. CLASSICAL GAIN SCHEDULING

In the classical gain scheduling approach [2], [12] also called *"linearization based gain scheduling"*. The first step is the selection of scheduling variable, then the nonlinear plant is linearized about a family of operating points or equilibrium points for fixed values of the scheduling variable. Noting that this assumption is then ignored and the scheduling variable is treated as time varying scheduling, that is measured and used to adjust the controller gains online.

consider the nonlinear plant described by [14]

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t)), t \geq 0 \end{cases} \tag{1}$$

where $x(t)$,$u(t)$ and $y(t)$ are the state vector, the input vector and the measured output vector respectively. It is assumed that there exists a family of equilibrium points (x_{eq}, u_{eq}) such that

$$0 = f(x_{eq}(\rho), u_{eq}(\rho)) \tag{2}$$

where ρ is the scheduling variable and is assumed to be bounded and measured during the control operation. For a fixed value of ρ, the following linearization family is obtained

$$\begin{cases} \dot{\tilde{x}}(t) = A(\rho)\tilde{x}(t) + B(\rho)\tilde{u}(t) \\ \tilde{y}(t) = C(\rho)\tilde{x}(t) \end{cases} \tag{3}$$

where

$$A(\rho) = \frac{\partial f(x_{eq}(\rho), u_{eq}(\rho))}{\partial x} \tag{4}$$

$$B(\rho) = \frac{\partial f(x_{eq}(\rho), u_{eq}(\rho))}{\partial u} \tag{5}$$

$$C(\rho) = \frac{\partial h(x_{eq}(\rho), u_{eq}(\rho))}{\partial x} \tag{6}$$

$$\tilde{x}(t) = x(t) - x_{eq}(\rho) \tag{7}$$

$$\tilde{u}(t) = u(t) - u_{eq}(\rho) \tag{8}$$

$$\tilde{y}(t) = x(t) - h(x_{eq}(\rho)) \tag{9}$$

Then any linear control design method can be applied resulting the collection of controllers denoted by

$$\begin{cases} \dot{\tilde{v}}(t) = \overline{A}(\rho)v(t) + \overline{B}(\rho)\tilde{y}(t) \\ \tilde{u}(t) = \overline{C}(\rho)v(t) + \overline{D}(\rho)\tilde{y}(t) \end{cases} \tag{10}$$

A. The Problem Of Classical Gain Scheduling

When endogenous signals such as state variables or system output are used as scheduling variables, hidden coupling terms (HCTs) appear in the linearized gain scheduling controller, indicating a connection between the plant and the controller caused by the fact that both the plant and the controller are scheduled by the same scheduling signal. [12].

Removing the HCTs from the design results in a lack of compatibility between the set of LTI controller dynamics utilized for design and the linearized dynamics of a nonlinear gain-scheduled controller. When the gain-scheduled controller is applied to the original nonlinear system, this mismatch will obviously result in substantial performance deterioration and potentially closed-loop system instability. Many techniques are used to overcome the problem of HTCs such as the exact cancellation of the HTCs [12]. Another technique consists of employing a velocity-based algorithm which is a specific gain-scheduled controller implementation to avoid this problem proposed in [2], there are other solutions that can fail in some cases or leads to more complex controller stucture [15]. A simpler and more general design based on eigenstructure assignment proposed in [16], [17].

IV. LPV BASED GAIN SCHEDULING

A. Linear Parametr Varying Systems (LPV) Representation

Given an LPV plant denoted by

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) = C(\rho(t))x(t) + D(\rho(t))u(t) \end{cases} \tag{11}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector, $y(t) \in \mathbb{R}^q$ is the output vector. Scheduling variables ρ is a time-varying vector such that $\rho(t) \in \mathcal{P} \subset \mathrm{R}^{n_\rho}$ where $\mathcal{P}$ is a given compact set, and the matrices $A(.)$,$B(.)$,$C(.)$ and $D(.)$ are parameter dependent matrices of appropriate dimensions of the scheduling signal vector ρ. One can also consider LPV systems with constraints on the rate of change $\dot{\rho}(t)$ in this case we define

$$|\dot{\rho}_i| < \overline{v_i}, i = 1, ..., n_\rho \tag{12}$$

And we define the set $\mathcal{V}$ such that

$$\mathcal{V} = \{v \in \mathbb{R}^{n_\rho} | v_i| < \overline{v_i}, i = 1, ..., n_\rho\} \tag{13}$$

B. Stability of LPV Systems

In this section we give an overview on the stability of LTI systems, then we derive the notions of stability for LPV systems, we have to mention that the LPV approach is very similar to robust analysis as we can see in figure 1

1) Stability Of LTI Systems: consider the following autonomous LTI system

$$\begin{cases} \dot{x}(t) = Ax(t), t \geq 0 \\ x(0) = x_0 \end{cases} \tag{14}$$

Theorem 1: **Lyapunov stability for LTI systems** The LTI system (14) is stable if and only if, there exits a symmetric matrix $P > 0$ such that

$$A^T P + PA < 0. \tag{15}$$

2) Stability Of LPV Systems Using Fixed Lyapunov function:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t), t \geq 0 \\ x(0) = x_0 \end{cases} \tag{16}$$

Theorem 2: The LPV system (16) is stable over $\mathcal{P}$ if, there exits a symmetric matrix $P > 0$ such that

$$A(\rho)^T P + PA(\rho) < 0.\forall \rho \in \mathcal{P} \tag{17}$$

3) Stability Of LPV Systems Using Parameter dependent Lyapunov function:

Theorem 3: The LPV system (16) is stable over $(\mathcal{P} \times \mathcal{V})$ if, there exits a symmetric,continuously differentiable matrix $P(\rho)$ such that

$$P(\rho) > 0 \forall \rho \in \mathcal{P} \tag{18}$$

and

$$A(\rho)^T P(\rho) + P(\rho)A(\rho) + \sum_{i=1}^{n_\rho} \dot{\rho}_i \frac{\partial P}{\partial \rho_i} < 0.\forall (\rho, \dot{\rho}) \in (\mathcal{P} \times \mathcal{V}) \tag{19}$$

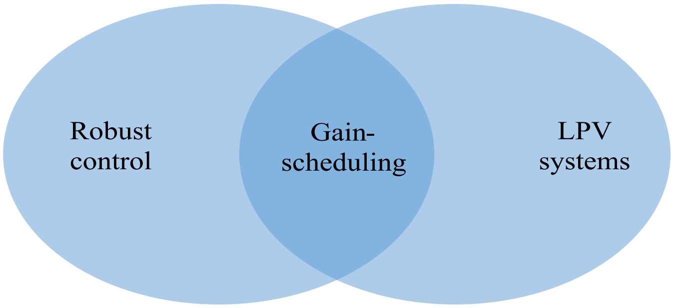

Fig. 1. Gain scheduling dependencies [18]

C. Performance of LPV Systems

Definition 5: **(Induced $\mathcal{L}_2$ norm)** [10] The induced $\mathcal{L}_2$ norm for the LPV system (11) with zero initial conditions is defined as:

$$\|T_{yu}\|_{i,2} = \sup_{\rho(t)\in\Omega} \sup_{u(t)\neq 0 \in \mathcal{L}_2} \frac{\|y\|_2}{\|u\|_2} \tag{20}$$

Where T_{yu} is the transfer between the output and the input that provides the forced response to an input signal $u(t) \in \mathcal{L}_2$ for zero initial conditions. a bound $\gamma > 0$ on $\|T_{yu}\|_{i,2}$ means that

$$\int_0^\infty y(\chi)^T y(\chi) \mathrm{d}\chi < \gamma^2 \int_0^\infty u(\chi)^T u(\chi) \mathrm{d}\chi \tag{21}$$

Notice that for a fixed parameter, the $\mathcal{L}_2$-norm equals the ∞-$norm$.

$$\|T_{yu}(\mathrm{i}\omega)\|_\infty = \sup_\omega \bar{\sigma}\left(T_{yu}(\mathrm{i}\omega)\right) \tag{22}$$

When $\|T_{yu}\|_{i,2} < \gamma$ holds and the LPV system (11) is exponentially stable, and γ is the performance level. The Bounded Real Lemma is extended in the following way to provide adequate criteria for evaluating performance by solving an optimisation problem with linear matrix inequality (LMI) constraints.

Theorem 4: Given the LPV system (11) with $(\psi, \dot{\psi}) \in \Psi \times \mathcal{V}$ There exists a positive definite differentiable symmetric matrix function $X(\psi)$ and

$$\begin{bmatrix} \dot{X}(\psi) + A^T(\psi)X(\psi) + X(\psi)A(\psi) & * & * \\ B^T(\psi)X(\psi) & -\gamma I_{n_u} & * \\ C(\psi) & D(\psi) & -\gamma I_{n_y} \end{bmatrix} < 0 \tag{23}$$

for all $(\psi, \dot{\psi}) \in \Psi \times \mathcal{V}$. Then,

1) $A(\psi)$ is parametrically-dependent quadratically (PDQ) stable over Ψ,

2) There exists a scalar φ with $0 \leq \varphi < \gamma$ such that $\|T_{yu}\|_{i,2} \leq \varphi$.

Where ψ refers to the scheduling variable $\rho(t)$.

D. Gain Schedued control Of an LPV System

The main step came after stability results and analysis in the LPV based frameworke for nonlinear systems is control design, which is directly derived form the theory of robust control. The main difference between them is that the scheduling variable of LPV system is available online to adjust the control law which leads to less conservative solutions, that is not the case for the uncertainties or unknown parameters in robust control theory that may be more conservative.

1) Gain-Schedued state feedback [5]: Gain-Schedued State Feedback controller is the simplest control design law,it has the following form

$$u(t) = K(\rho(t))x(t) \tag{24}$$

This controller's design is easy but needs all the states of the system to be available for measurement in order to be implemented, which is not always the case in many practical situations. [19].

2) Gain-Schedued static output feedback: The gain scheduling static output feedback controller is another basic gain scheduled control law, which is in the form

$$u(t) = K(\rho(t))y(t) \tag{25}$$

For more details the reader is referred to some new papers [20], [21]

3) Gain-Schedued observer-based feedback: in case where the state vector is not fully available for state feedback controller and the static output feedback cannot provide the design requirements one can use gain-scheduled observer-based control laws. The form of this controller is as follows

$$\begin{cases} \dot{\hat{x}}(t) = A(\rho(t))\hat{x}(t) + B(\rho(t))u(t) - L(\rho(t))(y(t) - \hat{y}(t)) \\ u(t) = C(\rho(t))\hat{x}(t) \end{cases} \tag{26}$$

$\hat{x}(t)$,$\hat{y}(t)$ are the estimated state and estimated output respectively, $L(\rho(t))$is the observer gain matrix. The gain scheduled controller is constructed using

$$u(t) = K(\rho(t))\hat{x}(t) \tag{27}$$

new results are presented in the work of Alam *et al* [22] and Sato [23]. There are also some details in [18] and references therein.

4) Gain-Schedued Dynamic Output Feedback [6]: The gain scheduled dynamic output feedback (DOF) leads to a less conservative controller, in contrast to other controllers,its structure is quite similar to observer based controllers but in this this case the state is not to be estimated, it takes the following form

$$\begin{cases} \dot{x}_(t) = A_c(\rho(t))x_c(t) + B_c(\rho(t))y(t) \\ u(t) = C_c(\rho(t))x_c(t) + D_c(\rho(t))y(t) \end{cases} \tag{28}$$

An increasing interest is given to GSDOF in many applications, in wind turbine control [11], in steering control for autonomous vehicles [8] and applied to vehicle stability control [24]. Theoretical results could be found in [25] and [26].

E. Reducing to finite number of LMIs

The fact that the solution involves solving an infinite number of linear matrix inequalities (LMIs) owing to the parameter space, is a significant difficulty in the formulation of LPV control problem. The problem has been reduced to a fixed number of LMIs via a variety of strategies: The polytopic approache [7], gridding-based approache [5] and LFT based approach [27]

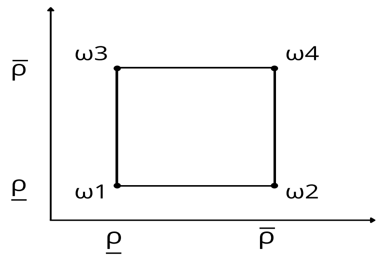

Fig. 2. Polytope of admissible parameters range

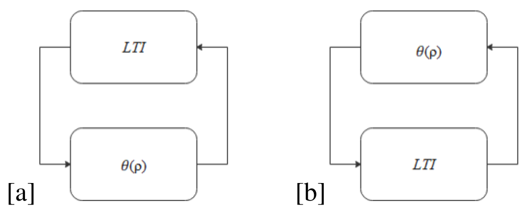

Fig. 3. a. Lower LFR b. Upper LFR reprsentation

Polytopic LPV synthesis: one can use this approach where the LPV system can be represented by state space matrices $A(\rho(t)), B(\rho(t)), C(\rho(t))$ and $D(\rho(t))$ where the scheduling parameter ranges within a fixed polytope as shown in figure 2 (i.e it can be expressed as convex hull),and the matrices $A(.), B(.), C(.)$ and $D(.)$ depend affinly on $\rho(t)$ such that:

$$\rho \in \text{Co}\{\omega_1, \ldots, \omega_m\} \tag{29}$$

and

$$\begin{pmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{pmatrix} = \sum_{j=1}^{m} \alpha_j(\rho) \begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix} \tag{30}$$

$$\alpha_j \geqslant 0, and \sum_{j=1}^{m} \alpha_j(\rho) = 1. \tag{31}$$

Gridding-based LPV synthesis: This approach can be used in the case where there is no affine parameter dependency (i.e. nonlinear) of the model. The gridding process consists of defining a subset of gridded parameters denoted $\mathcal{P}_g \in \mathcal{P}$, the controller is then synthesized by solving Linear Matrix Inequalities (LMIs) $\forall \rho \in \mathcal{P}_g$, and checking the LMI constraints in the selected grid density. Careful consideration must be given while choosing the grid density, a coarse grid may not correctly capture the parameter variation. On the other hand, solving LMIs for a dense grid can result in computing expenses and numerical problems. [28].

LFT synthesis: the LPV systems can be represented in the LFT-form, describes the interconnection between two subsystems, defined as a lower or upper linear fractional representation (LFR) between a nominal LTI model and a parameter-varying block as shown in figure 3.

V. Application to a rotational link

To conclude this work, we present a simulation example of classical GS and LPV gain-scheduled state feedback, applied to a simplified rotational link. This example is taken from [29]. Given the dynamics of the rotational link

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = mgLsin(x_1(t))/J - cx_2(t)|x_2(t)| + u(t) \end{cases} \tag{32}$$

Where $x_1 = \theta$ and $x_2 = \dot{\theta}$. Let $w(t)$ be the reference tracking for $\theta(t)$.

1) Classical gain scheduling design: In the classical gain scheduled controller, the nonlinear model of the plant is linearized about a fixed operating point, designing a proportional derivative feedback pole placement, then the gain scheduled control law is obtained when the scheduling variable is treated as time varying. Linearizing the system around an equilibrium leads to

$$\dot{\tilde{x}}(t) = A(\rho)\tilde{x}(t) + B(\rho)\tilde{u}(t) \tag{33}$$

$$x_{eq}(\rho) = \begin{pmatrix} \rho \\ 0 \end{pmatrix} \tag{34}$$

$$u_{eq}(\rho) = -mgLsin(\rho)/J \tag{35}$$

$$A(\rho) = \begin{pmatrix} 0 & 1 \\ mgLcos(\rho)/J & 0 \end{pmatrix}, B(\rho) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{36}$$

putting all parameters equal to one and for a fixed ρ, designing a proportional derivative feedback placing the closed loop poles at $-1 \pm j$

$$\tilde{u}(t) = -\begin{pmatrix} cos(\rho) + 2 & 2 \end{pmatrix} \tilde{x}(t) + 2\tilde{w}(t) \tag{37}$$

and leads to zero steady-state error to step commands, where $\tilde{w}(t) = w(t) - \rho$. The final gain scheduled control law is given by

$$u(t) = u_{eq}(\rho) - \begin{pmatrix} cos(\rho) + 2 & 2 \end{pmatrix} (x(t) - x_{eq}(\rho)) + 2(w(t) - \rho) \tag{38}$$

2) LPV gain-scheduled state feedback design: Now designing an LPV gain-scheduled state feedback controller using the approach proposed by Wu [5] on the same previous example. In the LPV GS controller a quasi-LPV representation is obtained by linearizing the nonlinear model of the system around a moving operating point. Available convex optimization tools make the control synthesis problem solvable, and allow us to design a gain-scheduled state-feedback controller using a gridding-based technique.

The LPV representation is given by

$$\begin{cases} \dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) \\ y(t) = C(\rho)x(t) \end{cases} \tag{39}$$

where $C(\rho) = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $A(\rho)$,$B(\rho)$ are the same in (36). $x_1 \in [0, 180], |x_2| < 10\frac{rad}{s}$. The Lyapunov matrix is selected arbitrary to be

$$P(\rho) = P_1 + P_2 x_2 \tag{40}$$

Weighting filters for mixed sensitivity S/KS shown in figure 4 are chosen as

$$W_S = \frac{1}{s}, \quad W_K = 0.05 \tag{41}$$

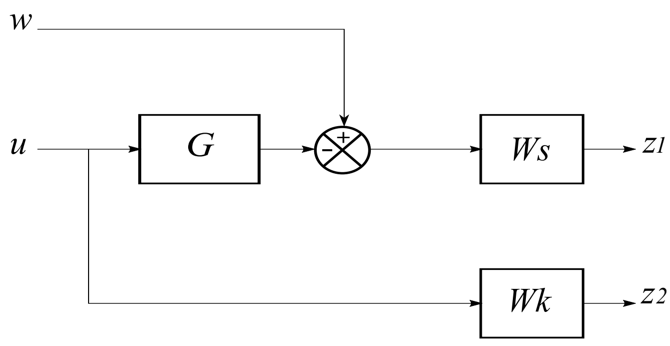

Fig. 4. Openloop S/KS

The augmented LPV plant is in the following form

$$\begin{bmatrix} \dot{x} \\ \hline z_1 \\ z_2 \end{bmatrix} = \left[\begin{array}{c|cc} A(\rho) & B_w(\rho) & B_u(\rho) \\ \hline C_1(\rho) & 0 & 0 \\ C_2(\rho) & 0 & I \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u \end{bmatrix} \qquad (42)$$

Then if there exists

$$P > 0 \qquad (43)$$

And

$$\begin{bmatrix} \Xi & PC_1^T & \frac{1}{\gamma}B_w \\ C_1 P & -I & 0 \\ \frac{1}{\gamma}B_w^T & 0 & -I \end{bmatrix} < 0 \qquad (44)$$

With

$$\Xi = P\left(A - B_u C_2\right)^T + \left(A - B_u C_2\right)P - B_u B_u^T - \sum_{i=1}^{n_\rho} \frac{\partial P}{\partial p_i}\dot{\rho} \qquad (45)$$

Then the GS state feedback control law is expressed as

$$u(t) = F(\rho)x(t) \qquad (46)$$

Where

$$F(\rho) = -B_u^T P^{-1}(\rho) - C_2 \qquad (47)$$

The LMIs are implemented and solved in MATLAB using the software packages Yalmip [30] and SeDuMi [31].

3) Discussion of results: Simulation results for tracking a square reference for the rotational link are illustrated in figure 5- 7.

We can see that both controllers perform well in all the operating ranges, It is interresting to mention that the GS-LPV State-feedback gives better results than the classical GS in terms of time response and overshoot as we can see in figure 5.

VI. CONCLUSIONS AND FUTURE WORK

Throughout this paper, a brief review of gain scheduling design control is given. Its importance in many engineering application is explained. The gain scheduling is presented using the classical and LPV approach. Recent theoretical results related to it are also given briefly. A comparative study between classical gain scheduling and LPV gain-scheduled state feedback is developed. Simulation results illustrates that the GS-LPV state feedback is able to stabilize the system

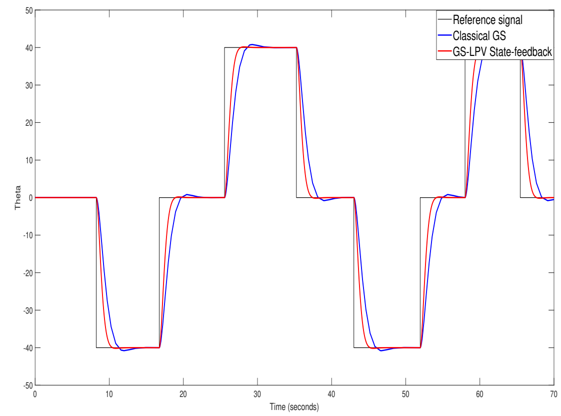

Fig. 5. Reference tracking

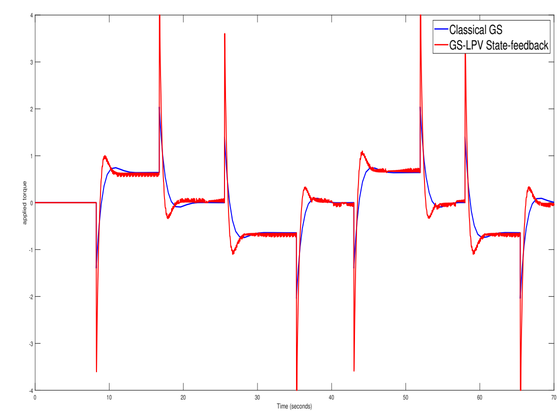

Fig. 6. Applied torque

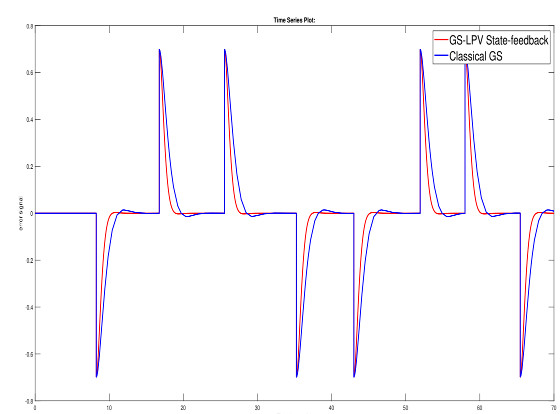

Fig. 7. Error signal

with higher performace than the classical GS. Our future work consists on designing gain scheduled controllers for more complex systems, with real world experiments.

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